Demand-side reactive strategies for supply disruptions in a multiple-product system

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ABSTRACT

This research examines demand-side reactive strategies for supply disruption in a multiple assemble-to-order system. We consider an assemble-to-order system with two substitute products where the demand is price-sensitive and disruption-sensitive. Two different supply disruption situations are examined: disruption of the low-value component and disruption of the high-value component. We propose and compare the performance of four reactive strategies for managing supply disruptions, namely, the backordering strategy, the upgrading/downgrading strategy, the compensation strategy, and the mixed strategy. We find that the compensation strategy and the mixed strategy can keep more customers than the backordering strategy and the upgrading strategy during the supply disruption of the low-value product. For the disruption of the high-value product, the total number of customers keeps constant. But it does lead to the reallocation of customers among the products. We find that the mixed strategy is the best reactive strategy and the backordering strategy is the worst one among the four reactive strategies.

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1. Introduction

As supply chains become more complex, disruption risks in supply networks have increased (Thun and Hoenig, 2011) and "the vulnerability of supply chains to disturbance or disruption has increased" (Christopher and Lee, 2004). Supplier bankruptcy, port stoppages, labor strikes, accidents and natural disasters, quality issues and machine breakdown (Finch, 2004; Sheffi, 2005), technological uncertainty and market thinness (Ellis et al., 2010) are possible causes for supply disruptions. It seems that supply disruptions occur more frequently and with more serious consequences (Wagner and Neshat, 2010). Supply disruptions can lead to excessive downtime of production resources, significant delays in customer deliveries, financial losses, and eventually a loss in the market value of the firm (Burke et al., 2007). A firm’s performance may drop sharply once the full impact of the disruption hits (Sheffi and Rice, 2005). For instance the disruptions of air transport in South-East Asian region caused by 9/11 attacks and a series of typhoons in 2001 resulted in $150 million loss for Compaq (Flower, 2001). Ericsson suffered a loss of 400 million Euros due to the supply disruption of Philips’s semiconductor plant in 2000 (Norman and Jansson, 2004).

In recent years, supply disruption management has gained considerable attention from both researchers and practitioners (Tang and Musa, 2011). Schmitt et al. (2010) demonstrate that supply disruptions can have significant negative impact on a company if it has not proactively protected itself against them. As the losses of supply disruptions can be huge, it is critical for companies to learn how to manage and control potential supply disruptions. Various strategies for managing supply disruptions have been considered, including multisourcing, flexibility, backup options and increasing buffer stock and capacity.

Zsidisin et al. (2000) conduct an analysis of in-depth interviews with purchasing professionals and they find that purchasing organizations often implement process-improvement and buffer strategies in response to supply risks. Li et al. (2010) investigate the impacts of supply disruption on the buyer’s sourcing strategy and the suppliers’ pricing strategy in a single-retailer two-supplier supply chain. Trkman and McCormack (2009) present preliminary research concepts regarding the identification and prediction of supply risk and provide a new method for the assessment and classification of suppliers based on their characteristics, performances and the environment of the industry in which they operate. Chopra and Sodhi (2004) identify several supply chain risk mitigation strategies, such as increase of capacity and adoption of flexibility and responsiveness. Sheffi and Rice (2005) list two basic approaches: building either redundancy or flexibility into the supply chain. They argue that redundancy is generally more costly because it involves adding safety stock, using multiple suppliers and maintaining slack in capacity utilization. Faisal et al. (2006) provide several enablers for supply chain disruption mitigation, including

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information sharing, supply chain agility, trust, collaborative relationship, etc. Tang (2006) presents nine different robust supply chain strategies that aim to enhance a firm's capability to sustain its operations when a major disruption hits. Tomlin (2006) investigates six countermeasures, both individually and in combinations: acquiring business interruption insurance, adding inventory, multiple-supplier sourcing, increased production or alternate route to market and demand management. Oke and Gopalakrishnan (2009) identify that better planning and coordination of supply and demand, flexible capacity, multiple sourcing, understanding and identification of supply chain vulnerability points and putting contingency plan in place are main strategies used to mitigate the supply-related risks in a retail supply chain. Yang et al. (2009) investigate how risk-management strategies of the manufacturer change and examine whether risk-management tools are more or less valuable in the presence of asymmetric information. They find that asymmetric information can have a pronounced effect on the manufacturer's risk-management strategy, and it may cause the manufacturer to stop using the backup production of a less reliable supplier, while continuing to use the backup production of a more reliable supplier.

There are papers focusing on disruption recovery. Xia et al. (2004) present a general disruption recovery planning approach for a two-stage production and inventory control system. Eisenstein (2005) addresses disruptions in an economic lot scheduling environment. He assumes that the original schedule is fixed and focuses on recovery after one or more shocks have occurred by introducing a new class of policies called dynamic produce-up-to policies. Yang et al. (2005) also focus on recovery of the production plan after disruption has occurred. They analyze the initial planning problem as a min-cost network flow problem and propose a dynamic programming algorithm to account for cost and demand disruptions under the recovery plan. Xiao and Yu (2006) present a raw material supply disruption recovery model and illustrate the effects of recovery of the raw material supply on the supply chains. Adhiyia et al. (2007) propose a model-based framework for rescheduling operations in the face of supply chain disruptions. Abdelghany et al. (2008) present an integrated decision support tool for airlines schedule disruption management which provides proactive recovery plans to enable near real-time response.

The majority of supply disruption management strategies presented in the supply disruption literature focus on supply-side management strategies. Demand-side management strategy can also be an effective way to react to disruption when the supply of a particular product is disrupted. Companies can use pricing mechanism and promotion to entice customers to choose products that are widely available when the supply of certain product is disrupted. For example, when Dell was facing supply disruptions from their Taiwanese suppliers after an earthquake in 1999, Dell immediately deployed a contingency plan by offering special “low-cost upgrade” options to customers if they chose similar computers with components from other suppliers. This dynamic pricing and promotion strategy enabled Dell to satisfy its customers during a supply disruption (Martha and Subbakrishna, 2002).

To our knowledge, there are few papers that focus on demand management strategy for supply disruption. This has motivated us to investigate the demand-side reactive strategies for supply disruption. To address the gap in the current literature, we investigate the comparison and selection of demand-side reactive strategies for supply disruption. We consider an assemble-to-order system with two substitute products (labeled A and B). Product A consists of one unit of each components \( a_0 \) and \( a_1 \), while product B consists of one unit each of components \( b_0 \) and \( b_1 \) (see Fig. 1). Products A and B are two options with different values or qualities. For simplicity, we assume product B has a higher quality (higher value) than product A. Firms often differentiate their product lines vertically to capture consumers’ differential willingness to pay for quality. For example, a typical desktop product line includes CPUs with clock speed ranging from 2.2 GHz to 2.8 GHz, memory from 256 MB to 2 GB, and so on (Draganska and Jain, 2006).

Technically, components \( a_0 \) and \( a_1 \) are designed to be the best match. Similarly, components \( b_0 \) and \( b_1 \) are designed to be the best match. Components \( a_1 \) and \( b_1 \) are technically substitutable. For simplicity, we suppose \( b_1 \) has a higher value (quality). Component \( b_1 \) and \( a_0 \) can be assembled to form an upgraded version of product A, which is called \( A_u \) (see Fig. 1). Components \( a_1 \) and \( b_0 \) can be assembled to form a downgraded version of product B, which is called \( B_d \) (see Fig. 1).

We define the potential aggregate demand \( D \) of all customers for product A and B during each single period as a random variable with a mean value of \( \mu \). Assume that there is a fixed demand intensity of potential customers arriving in each period and the potential aggregate demands in different period are independent and identically distributed. But the decision that an arriving customer makes depends on the product values and prices. We assume that each arriving customer in each period would buy only one unit of the products offered by the manufacturer or buy nothing.

![Fig. 1. The assembler-to-order system.](image-url)
Let $s_i, i=1,2$ be the value associated to products A and B representing the customer’s willingness to pay for the products. In the absence of price and delivery considerations, every customer prefers product B to product A because product B has a higher value. Similar to Mendelson and Parlaktürk (2008), and Shao and Ji (2009), we assume customers are heterogeneous. A customer of type-$i$ is willing to pay $b_i$ for a unit of each product A and B. Customer types are distributed uniformly on [0,1]. Let $p_i, i=1,2$ be the price of product A and B, and $c_i, i=1,2$ be the cost of product A and B, respectively.

The manufacturer purchases components from the suppliers and assembles products according to orders. We assume the system is totally order-driven. When supply disruption occurs, the manufacturer cannot acquire the corresponding components from the supplier on time. It takes $T$ periods for the supplier to recover from a disruption. We assume that the initial product prices are kept constant during the disruption due to some market regulations. Under supply disruption, the sequence of events is as follows. The supplier reports the disruption event and notifies the manufacturer the duration of the disruption. The manufacturer makes decisions on reactive strategies. At the beginning of each period during the disruption, customers arrive and make buying decisions according to the options offered by the manufacturer. We assume the manufacturer has a monopoly status. Competition from other manufacturers is not considered in this paper. We consider two different supply disruption situations: disruption of the low-value component $a_1$ and disruption of the high-value component $b_1$.

When the supply of component $a_1$ is disrupted, the value of product A with delayed delivery is reduced. Define $s_1 - \lambda_1 I$ as the value of product A with delayed delivery in period $t$. Here $\lambda_1$ represents customers’ sensitivity to delayed delivery of product A, and $I = T + 1 - t$ is the number of delayed periods due to the disruption. The manufacturers can offer customers an upgraded version of product A by substituting $b_1$ for $a_1$. We assume that the supply of $b_1$ is unlimited in this case. The value increase of the upgraded version is $\Delta s_1$. A customer only pays extra $\Delta p_1$ for upgrading product A. The manufacturer can also pay a compensation for delayed delivery to keep customers. Define $\lambda_1 I$ as the compensation offered by the manufacturer. Here $\lambda_1 I$ is the penalty per period that the manufacturer offers for delayed delivery of product A. In this situation, a customer actually pays $p_1 - \lambda_1 I$ for product A with delayed delivery.

Similarly, when the supply of component $b_1$ is disrupted, the value of product B with delayed delivery is reduced. Define $s_2 - \lambda_2 I$ as the value of product B with delayed delivery in period $t$. Here $\lambda_2$ represents customers’ sensitivity to delayed delivery of product B. The manufacturer can offer customers a downgraded version of product B by substituting $a_1$ for $b_1$. We also assume that the supply of $a_1$ is unlimited in this case. The value decrease of the downgraded version is $\Delta s_2$. A customer can get a rebate of $\Delta p_2$ for downgrading product B. The manufacturer can also adopt a compensation policy. Define $\lambda_2 I$ to be the compensation offered by the manufacturer. Here $\lambda_2 I$ is the penalty per period that the manufacturer offers for delayed delivery of product B. In this situation, a customer actually pays $p_2 - \lambda_2 I$ for product B with delayed delivery.

**Assumption 1.** The price per unit value of a high-value product is higher than that of a low-value product, i.e.

\[
\frac{P_2}{S_2} > \frac{P_1}{S_1}
\]

**Assumption 2.** The value of the upgraded version is lower than the high-value product, i.e., $s_2 > s_1 + \Delta s_1$. In addition,

\[
\frac{P_2}{S_2} > \frac{P_1 + \Delta P_1}{S_1 + \Delta S_1} > \frac{P_1}{S_1 - \lambda_1 I}
\]

**Assumption 3.** The value of the downgraded version is lower than the high-value product with delayed delivery and higher than the low-value product, i.e., $s_2 - \lambda_2 I > s_2 - \Delta s_2 > s_1$. In addition,

\[
\frac{P_2 - \lambda_2 I}{S_2 - \lambda_2 I} > \frac{P_2 - \Delta P_2}{S_2 - \Delta S_2} > \frac{P_1}{S_1}
\]

The above assumptions are rational and realistic. In reality, a firm makes super profits by charging high prices for high-value products. The assumptions imply positive return of investment in improving values or qualities. In reality, the price of a luxurious product usually grows at a faster rate than its value. We assume the duration of the supply disruption lasts a certain reasonable time and the value decrease due to delayed delivery is much smaller than the product value itself.

### 3. The optimal pricing in normal conditions

A customer would maximize his surplus (the difference between what he is willing to pay and the price charged) to decide his purchasing behavior. If the customer surplus is less than zero, then he would choose to buy none of the products. Thus, in the normal conditions each potential arriving customer decides his purchasing behavior. If the customer surplus is less than zero, then he would choose to buy none of the products. Let $m_{01}, i=1,2$ be the percentage of customers who would choose product A and B.

**Lemma 1.** Let $(s_1, p_1)$ and $(s_2, p_2)$ be the quality and price set of the two substitute products A and B in normal conditions. Then

\[
m_{01} = \frac{P_2 - P_1}{S_2 - S_1} \frac{P_1}{S_1}, \quad m_{02} = 1 - \frac{P_2 - P_1}{S_2 - S_1}.
\]

The proof of Lemma 1, as well as the proofs of other lemmas and propositions, is given in the Appendix.

Let $\pi_0(P)$ denote the expected profit of the system in each period in normal conditions. We have

\[
\pi_0(P) = E\left[ (p_1 - c_1) \left( \frac{P_2 - P_1}{S_2 - S_1} - \frac{P_1}{S_1} \right) D + (p_2 - c_2) \left( 1 - \frac{P_2 - P_1}{S_2 - S_1} \right) D \right].
\]

**Proposition 1.** There exist the optimal pricing decisions for the assemble-to-order system with two substitute products:

\[
p_1^* = \frac{S_1 + c_1}{2}, \quad p_2^* = \frac{S_2 + c_2}{2}.
\]

### 4. Reactive strategies for supply disruption of the low-value product

In case of supply disruption of the low-value component, the manufacturer cannot acquire component $a_1$ from the supplier and promise customers on-time delivery of product A. In this section, we analyze four different reactive strategies for supply disruption of product A, namely, backordering strategy, upgrading strategy, compensation strategy and mixed strategy.

#### 4.1. Backordering strategy

In a backordering strategy, the manufacturer passively accepts the disruption and backorders customers’ orders until the supplier recovers from the disruption. In this case, a customer who arrives in period $t$ for product A has to wait for extra $T + 1 - t$ periods. The surplus of a type-$i$ customer with delayed delivery of product A is given by

\[
U(\theta, I) = \theta(s_1 - \lambda_1 I) - p_1
\]
Here $l+T+1-t$ is the number of delayed periods due to the disruption, and $\lambda_1$ represents customers’ sensitivity to delayed periods.

Define $m_{1b}$, $m_{1u}$ as the percentage of customers who choose to place orders for product A and B, respectively, during the disruption when the manufacturer adopts the backordering strategy. We have the following lemma:

**Lemma 2.** Let $(s_1, s_2, p_1)$ and $(s_2, p_2)$ be the value and price set of the two substitute products in period $t$. Then

$$m_{1b} = \frac{p_2 - p_1}{s_2 - s_1 + \lambda_1} - \frac{p_1}{s_1 - \lambda_1}, \quad m_{1u} = 1 - \frac{p_2 - p_1}{s_2 - s_1 + \lambda_1}.$$

Define $p_b^*$ as the manufacturer’s expected profit in period $t$ during the disruption when a backordering strategy is adopted. The manufacturer has the following profit function:

$$p_b^* = E\left( \frac{p_1 - c_1}{s_1 - \lambda_1} \right) D + \left( \frac{p_2 - c_2}{s_2 - s_1 + \lambda_1} \right) D.$$ 

### 4.2. Upgrading strategy

In the upgrading strategy, the manufacturer takes marketing tactics to shift the demand from the disrupted low-value component to a high-value one. Upgrading strategy is a common reactive strategy which is usually adopted in build-to-order companies, such as Dell Computer. For example, if the supply of Sony 17-in. monitors is short, Dell could offer a 19-in. model at a lower price (Merritt, 2001). The supply disruption problem is partially solved by upgrading 17-in. monitors to 19-in. monitors.

In the case that the manufacturer adopts the upgrading strategy, some customers who arrive and want to buy product A may move to buy the upgraded version of product A. The surplus of a type-$\theta$ customer who chooses to upgrade product A is given by

$$U(\theta, \Delta p_1) = \theta(s_1 + \Delta s_1) - (p_1 + \Delta p_1).$$

Here $\Delta s_1$ is the value increase of the upgraded version, and $\Delta p_1$ is the price increase for upgrading product $A$.

Define $m_{1b}^u$, $m_{1u}^u$ and $m_{1u}^d$ as the percentage of customers who choose to place orders for product A, the upgraded version of product A and product B, respectively, in period $t$ during the disruption when the upgrading strategy is adopted. We have the following lemma:

**Lemma 3.** Let $(s_1, s_2, p_1)$, $(s_1 + \Delta s_1, p_1 + \Delta p_1)$ and $(s_2, p_2)$ be the value and price set of the products offered in period $t$ during the disruption when the upgrading strategy is adopted. Then

$$m_{1b}^u = \frac{\Delta p_1}{\Delta s_1 + \lambda_1} - \frac{p_1}{s_1 - \lambda_1}, \quad m_{1u}^u = \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1}, \quad m_{1u}^d = 1 - \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1}.$$ 

Define $p_b^u$ as the manufacturer’s expected profit in period $t$ during the disruption when an upgrading strategy is adopted. The manufacturer has the following profit function:

$$p_b^u = E\left[ \frac{\Delta p_1}{\Delta s_1 + \lambda_1} - \frac{p_1}{s_1 - \lambda_1} \right] D + \left( \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1} \right) D + \left( \frac{p_2 - c_2}{s_2 - s_1 + \lambda_1} \right) D.$$ 

Here $\Delta c_1$ is the cost increase of upgrading product A.

**Proposition 2.** The expected profit function in the upgrading strategy $p_b^u$ is concave in $\Delta p_1$. There exists the optimal special offer for the product upgradation:

$$\Delta p_1^* = \frac{\Delta c_1}{2} + \frac{(s_2 - s_1)(s_1 + \lambda_1)}{2(s_2 - s_1 + \lambda_1)}.$$ 

### 4.3. Compensation strategy

In the compensation strategy, the manufacturer pays a penalty to the customers for late delivery of product $A$. Customers actually pay $p_1 - z_1 l$ for each unit of product $A$. Here, $z_1$ is the penalty per delayed period that the manufacturer offers. The surplus of the disrupted product $A$ for a type-$\theta$ customer in the compensation strategy is given by

$$U(\theta, z_1 l, l) = \theta(s_1 - \lambda_1) - (p_1 - z_1 l).$$

Define $m_{1b}^c$, $m_{1u}^c$ and $m_{1u}^d$ as the percentage of customers who choose to place orders for product A and B, respectively, in period $t$ during the disruption when the compensation strategy is adopted. We have the following lemma:

**Lemma 4.** Let $(s_1, s_2, p_1, p_2)$ and $(s_2, p_2)$ be the value and price set of the products offered in period $t$ during the disruption when the compensation strategy is adopted. Then

$$m_{1b}^c = \frac{p_2 - p_1 + z_1 l}{s_2 - s_1 + \lambda_1}, \quad m_{1u}^c = \frac{p_1 - z_1 l}{s_1 - \lambda_1}, \quad m_{1u}^d = 1 - \frac{p_2 - p_1 + z_1 l}{s_2 - s_1 + \lambda_1}.$$ 

Define $p_b^c$ as the manufacturer’s expected profit in period $t$ during the disruption when a compensation strategy is adopted. The manufacturer has the following profit function:

$$p_b^c = E\left[ \left( \frac{p_2 - p_1 + z_1 l}{s_2 - s_1 + \lambda_1} - \frac{p_1 - z_1 l}{s_1 - \lambda_1} \right) D + \left( \frac{p_2 - p_1 + z_1 l}{s_2 - s_1 + \lambda_1} \right) D \right]$$

**Proposition 3.** The expected profit function in the compensation strategy $p_b^c$ is concave in $z_1$. There exists the optimal compensation rate for the delayed delivery of the low-value product A

$$z_1^* = \frac{\lambda_1}{2}.$$ 

### 4.4. Mixed strategy

In the mixed strategy, the manufacturer offers customers a menu of choices when the supply of the low-value component is disrupted. Each arriving potential customer has a menu of choices, i.e., buying the high-value product, buying an upgraded version of the low-value product, ordering the low-value product and getting a compensation for late delivery, or leaving without buying anything.

Define $m_{1b}^m$, $m_{1u}^m$ and $m_{1u}^d$ as the percentage of customers who choose to place orders for product A, the upgraded version of product A and product B, respectively, in period $t$ during the disruption when a mixed strategy is adopted. We have the following lemma:

**Lemma 5.** Let $(s_1, s_2, p_1, p_2)$ and $(s_2, p_2)$ be the value and price set of the products offered in period $t$ during the disruption when a mixed strategy is adopted. Then

$$m_{1b}^m = \frac{\Delta p_1 + z_1 l}{s_1 + s_2 + \lambda_1}, \quad m_{1u}^m = \frac{p_2 - p_1 - \Delta p_1 - z_1 l}{s_2 - s_1 - \Delta s_1}, \quad m_{1u}^d = 1 - \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1}.$$ 

Define $p_b^m$ as the manufacturer’s expected profit in period $t$ during the disruption when a mixed reactive strategy is adopted.
The manufacturer has the following profit function:

\[ p_t^m = E \left[ \frac{\Delta p_1 + \Delta l}{\Delta s_1 + \Delta l} \left( \frac{p_1 - p_2}{s_1 - s_2} \right) \right] \]

\[ + \left( \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1} \right) \left( \frac{\Delta p_1 + \Delta l - c_1}{\Delta s_1 + \Delta l} \right) \]

\[ + \left( 1 - \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1} \right) \left( p_2 - c_2 \right) \]

**Proposition 4.** The expected profit function in the mixed reactive strategy \( p_t^m \) is joint-concave in \( \Delta p_1 \) and \( x_t \). There exist the optimal compensation and upgrading pricing decisions:

\[ \Delta p_t^* = \frac{\Delta s_1 + \Delta c_1}{2}, \quad x_t^* = \frac{\lambda_t}{2}. \]

4.5. Comparison of reactive strategies for disruption of low-value product

In this section, we consider the performance of the four different reactive strategies discussed above.

According to Lemmas 2–5, we find that

\[ \sum m_i^t = \sum m_iw^t = 1 - \frac{s_1 + c_1}{2(s_1 - \lambda t)} < \sum m_o^t = \sum m_o^i \]

\[ = 1 - \frac{s_1 + c_1 - \lambda t}{2(s_1 - \lambda t)} \]

The results show that some customers will be lost in the disruption no matter what reactive strategies are adopted. But the compensation strategy and the mixed strategy can keep more customers than the backordering strategy and the upgrading strategy. The number of lost customers in the reactive strategies is determined by \( \lambda_t \) and \( t \). More customers are lost at the beginning of the disruption with a high time-sensitive demand (see Fig. 2).

We further find that \( m_o^t < m_i^t \) and \( m_i^o > m_i^2 > m_o^2 \), which implies that more customers who originally plan to buy the low-value product will turn to buy a high-value product or leave the system without buying anything when a backordering strategy is adopted. In contrast, a compensation strategy can keep more customers for the disrupted low-value product.

We also find that \( m_i^o < m_i^o, m_iw = m_i^w \) and \( m_i^2 > m_o^2 \). It implies that there is an equal number of customers who would choose the upgraded version in both the upgrading strategy and the mixed strategy. But the mixed strategy can keep more customers for the disrupted low-value product.

**Proposition 5.** The compensation strategy and the upgrading strategy are always superior to the backordering strategy during the disruption of the low-value product.

**Proposition 6.** The mixed strategy is always superior to the compensation strategy and the upgrading strategy during the disruption of the low-value product.

**Propositions 5 and 6** show that the mixed strategy is the best reactive strategy and the backordering strategy is the worst one among the four reactive strategies for the disruption of the low-value product.

5. Reactive strategies for supply disruption of the high-value product

In case of supply disruption of the high-value component, the manufacturer cannot acquire component \( b_1 \) from the supplier and promise customers on-time delivery of product B. The corresponding reactive strategies are backordering, downgrading, compensation, and the mixed strategies.

In the backordering strategy, the surplus of a type-\( \theta \) customer arriving in period \( t \) for product B with delayed delivery is given by

\[ U(t, l) = \theta(s_2 - \Delta l) - p_2 \]

Here, \( \lambda_2 \) represents customer’s sensitivity to delayed periods of the high-value product.

In the downgrading strategy, some customers who arrive for the high-value product B would move to the downgraded version of product B, and some would move to the low-value product A. The surplus of a type-\( \theta \) customer who choose the downgraded version of product B is given by

\[ U(t, \Delta l) = \theta(s_2 - \Delta l) - (p_2 - \Delta p_2) \]

Here \( \Delta s_2 \) is the value decrease of the downgraded version of product B, and \( \Delta p_2 \) is the price rebate for customers who move from the high-value product B to the downgraded version of product B.

Similar to the compensation strategy for the low-value product disruption, the manufacturer pays a penalty to customers for late delivery of the high-value product. Customers who buy product B actually pay \( p_2 - \lambda_2 l \) for each unit of the product. Here \( \lambda_2 \) is the penalty per delayed period for late delivery of product B.
The surplus of the disrupted product B for a type-$\theta$ customer is given by
\[ U(\theta, \tilde{x}_B, t) = \theta(s_2 - \tilde{x}_2) - (p_2 - \tilde{x}_2) \]

In the mixed strategy, the manufacturer offers a compensation for those customers who are willing to wait for the high-value product, and provide a downgraded version of the high-value product with a price rebate for those who are unwilling to wait.

Define $m_{i1}^t$, $m_{i2}^t$ and $m_{i3}^t$, $i=a,b,c,d,m$, as the percentage of customers who choose to place orders for product A, the downgraded version of product B and product B, respectively, in period $t$ during the disruption when the manufacturer adopts different strategies.

We have the following lemma:

**Lemma 6.**

1. When the backordering strategy is adopted, then
   
   \[ m_{i1}^t = \frac{p_2 - p_1}{s_2 - s_1 - \tilde{x}_2} - \frac{p_1}{s_1} \quad m_{i2}^t = 1 - \frac{p_2 - p_1}{s_2 - s_1 - \tilde{x}_2} \]

2. When the downgrading strategy is adopted, then
   
   \[ m_{i1}^t = \frac{p_2 - \Delta p_2 - p_1}{s_2 - \Delta s_2 - s_1} - \frac{p_1}{s_1} \quad m_{i2}^t = \frac{\Delta p_2}{s_2 - \Delta s_2 - s_1} \]

3. When the compensation strategy is adopted, then
   
   \[ m_{i1}^t = \frac{p_2 - p_1 - \tilde{x}_2}{s_2 - s_1 - \tilde{x}_2} - \frac{p_1}{s_1} \quad m_{i2}^t = 1 - \frac{p_2 - p_1 - \tilde{x}_2}{s_2 - s_1 - \tilde{x}_2} \]

4. When the mixed strategy is adopted, then
   
   \[ m_{i1}^t = \frac{p_2 - \Delta p_2 - p_1}{s_2 - \Delta s_2 - s_1} - \frac{p_1}{s_1} \quad m_{i2}^t = \frac{\Delta p_2}{s_2 - \Delta s_2 - s_1} \]

Define $\pi_d^t$ as the manufacturer’s expected profit in period $t$ during the disruption when a downgrading strategy is adopted. The manufacturer has the following profit function:

\[ \pi_d^t = E \left[ \left( \frac{p_2 - \Delta p_2 - p_1}{s_2 - \Delta s_2 - s_1} - \frac{p_1}{s_1} \right) \left( p_1 - c_1 \right) D + \left( \frac{\Delta p_2}{s_2 - \Delta s_2 - s_1} \right) \left( p_2 - \Delta p_2 - p_1 \right) \right] \]

Define $\pi_c^t$ as the manufacturer’s expected profit in period $t$ during the disruption when a compensation strategy is adopted. Then,

\[ \pi_c^t = E \left[ \left( \frac{p_2 - p_1 - \tilde{x}_2}{s_2 - s_1 - \tilde{x}_2} - \frac{p_1}{s_1} \right) \left( p_1 - c_1 \right) D + \left( \frac{\Delta p_2}{s_2 - \Delta s_2 - s_1} \right) \left( p_2 - p_1 - \tilde{x}_2 \right) \right] \]

The expected profit function $\pi_c^t$ is concave in $\tilde{x}_2$. There exists the optimal compensation rate for delayed delivery of the high-value product: $\tilde{x}_2^* = \tilde{x}_2^*/2$.

Define $\pi_m^t$ as the manufacturer’s expected profit in period $t$ during the disruption when a mixed strategy is adopted. Then,

\[ \pi_m^t = E \left[ \left( \frac{p_2 - \Delta p_2 - p_1}{s_2 - \Delta s_2 - s_1} - \frac{p_1}{s_1} \right) \left( p_1 - c_1 \right) D + \left( \frac{\Delta p_2}{s_2 - \Delta s_2 - s_1} \right) \left( p_2 - \Delta p_2 - p_1 \right) \right] \]

The results show that the disruption of the high-value product does not lead to the loss of customers in the system. But it does lead to the reallocation of customers among the products. From Lemma 6, we know that $m_{i1}^t > m_{i2}^t$ and $m_{i3}^t < m_{i2}^t$, which implies that more customers who originally plan to buy the high-value product will turn to buy the low-value product when a backordering strategy is adopted. In contrast, a compensation strategy can keep more customers for the disrupted high-value product.

We further find that $m_{i1}^t > m_{i1}^t$, $m_{i2}^t = m_{i2}^t$ and $m_{i2}^t < m_{i2}^t$, which means that the mixed strategy can keep more customers for the disrupted high-value product, and there is an equal number of customers who will choose the downgraded version in both the downgrading strategy and the mixed strategy.

We have the following proposition considering the profit performance of the four reactive strategies for disruption of product B discussed above:

**Proposition 7.** The compensation strategy and downgrading strategy for disruption of high-value product are always superior to the backordering strategy. And the mixed strategy is always superior to the compensation strategy and the downgrading strategy.

**Proposition 7** shows that the mixed strategy is the best reactive strategy and the backordering strategy is the worst one among the four reactive strategies for supply disruption of the high-value product.

### 6. Numerical examples

In this section, we analyze numerical examples to compare the performance of reactive strategies for supply disruption management of substitute products. We consider the following basic system parameters: $s_1 = 12$, $s_2 = 18$, $c_1 = 4$, $c_2 = 8$, $\mu = 1000$, $T = 12$, $\Delta s_1 = 3$, $\Delta s_2 = 3$.

**Fig. 3** shows percentage of customers in each period during the supply disruption of the low-value product when different reactive strategies are adopted. We find that the number of customers for the disrupted low-value product decreases, while the number of customers for the substitute high-value product increases in both the backordering strategy and the compensation strategy. More customers will turn to the substitute high-value product at the beginning of the disruption. And more customers will turn to the substitute high-value product in the backordering strategy than those in the compensation strategy (see **Fig. 3a**).

The number of customers for the substitute high-value product increases in the upgrading strategy at the beginning of the disruption. But it decreases in $t$. Some customers for the high-value product tend to turn to the upgraded version of the low-value product in the end of the disruption. We find that the upgraded version of product A could attract some customers from the segment of the high-value product. More customers will choose to the high-value product or the upgrade version of the low-value product at the beginning of the disruption in the upgrading strategy.
More customers will choose the upgraded version of the low-value product at the beginning of the disruption in the mixed strategy. We find the number of customers moving from the low-value product to the upgraded version in both the upgrading strategy and the mixed strategy are equal (see Fig. 3b).

Fig. 4 shows the impact of customer’s time sensitivity to the low-value product disruption on percentage of customers. It shows that the number of customers who turn to the high-value product in the backordering strategy and the compensation strategy increases in $\lambda_1$. And the number of customers who choose the...
upgraded version of the disrupted low-value product increases in $\lambda_1$ in both the upgrading strategy and the mixed strategy (see Fig. 4b).

And as $\lambda_1$ increases, more customers would turn to the high-value product in the upgrading strategy, but the number of customers for the high-value product keeps constant in the mixed strategy.

We find that the profits in the four reactive strategies increase in $t$, which implies that the impact of disruption on the manufacturer's profit is gradually reduced as time elapses. The profits in the mixed strategy are always higher than those in the other three reactive strategies (see Fig. 5), which implies that the mixed strategy is the best one among the four reactive strategies. And the profits in the backordering strategy are always lower than those in the other three strategies (see Fig. 5), which implies that the backordering strategy is the worst one among the four strategies. Fig. 5 also shows that the upgrading cost $\Delta c_1$ and customer's sensitivity to the disruption of the low-value product are two main parameters that impact the performance of the upgrading strategy and the compensation strategy. The compensation strategy is superior to the upgrading strategy when $\Delta c_1$ and $\lambda_1$ are relatively large (see Fig. 5a). Otherwise, the upgrading strategy may be preferred (see Fig. 5b). We also find that the profits in the four reactive strategies decrease in $\lambda_1$ (see Fig. 6).

When the supply of the high-value product is disrupted, the total number of customers keeps constant. The number of customers for the disrupted high-value product decreases and the number of customers for the substitute low-value product increases in both the backordering strategy and the compensation strategy. Fig. 7a shows that $m_{t1}$ and $m_{t2}$ decrease in $t$, which implies that more customers will turn to the low-value product at the beginning of the disruption. Fig. 7a also shows that more customers will turn to the low-value product in the backordering strategy than those in the compensation strategy. The number of customers for the low-value product keeps constant in the mixed strategy. Fig. 7b shows that $m_{t1}$, $m_{t2}$, $m_{m2}$ decrease in $t$ and $m_{d2}$, $m_{c2}$ increase in $t$, which implies that more customers will turn to the downgraded version of the high-value product in the mixed strategy, and will turn to the downgraded version or the low-value product in the downgrading strategy at the beginning of the disruption of the high-value product.

We find that the downgraded version of product B could attract part of customers from the segment of the low-value product in the mixed strategy. And it may also attract some customers from the low-value product segment in the end of the disruption in the downgrading strategy (see Fig. 7b). We find that customer's sensitivity to the supply disruption of the high-value product influences customer's choices. As $\lambda_2$ increases, more customers would turn to the low-value product or the downgraded version of high-value product (see Fig. 8).

Fig. 9 shows that the downgrading cost $\Delta c_2$ and customer's sensitivity to the disruption of the high-value product are two main parameters that impact the performance of the downgrading strategy and the compensation strategy. The downgrading strategy is preferred when $\Delta c_2$ is relatively high and $\lambda_2$ is relatively low (see Fig. 9b). Otherwise, the compensation strategy is preferred (see Fig. 9a). Similar to the result in the supply disruption of the low-value product, $\lambda_2$ has a decreasing impact on profits in the supply disruption of the high-value product (see Fig. 10).

7. Conclusions

When designing and adopting a reactive strategy to manage demand under supply disruption, the manufacturer must learn how the strategy would affect the expected sales revenue. Essentially, an established robust supply chain strategy would enable a firm to deploy the associated contingency plans efficiently and effectively when facing a disruption. Therefore, having a robust supply chain strategy could make a firm become more resilient (Tang, 2006). This research examines demand-side reactive
strategies for supply disruption in a multiple assemble-to-order system. We consider an assemble-to-order system with two substitute products where the demand is price-sensitive and disruption-sensitive. Two different supply disruption situations are examined: disruption of the low-value component and disruption of the high-value component.

We compare the performance of four reactive strategies, namely, backordering strategy, upgrading/downgrading strategy, compensation strategy, and mixed strategy. We find that some customers will be lost in the disruption of the low-value product no matter what reactive strategies are adopted. This is due to the fact that the delayed delivery of the low-value product would reduce the reservation value of some customers and those customers whose surplus (the difference between customer reservation value and the price charged) becomes negative would leave without placing orders. But the compensation strategy and the mixed strategy can keep more customers than the backordering strategy and the upgrading strategy. More customers are lost at the beginning of the disruption with a high time-sensitive demand. The mixed strategy is the best reactive strategy and the backordering strategy is the worst one among the four reactive strategies for the disruption of the low-value product. The results for the disruption of the high-value product are all different. We find that the disruption of the high-value product does not lead to the loss of customers in the system. This is due to the fact that the disruption of the high-value product does not reduce the surplus of those who choose the low-value product and some customers who originally plan to buy the high-value product just turn to the low-value product or the downgraded version. Thus the total number of customers keeps constant. But it

![Graph](image_url)

**Fig. 8.** Impact of $\lambda_2$ on $m$ in reactive strategies for high-value product disruption. (a) Backordering strategy and compensation strategy. (b) Upgrading strategy and mixed strategy.

![Graph](image_url)

**Fig. 9.** Comparison of profits in reactive strategies for high-value product disruption. (a) $\Delta c_1=2, \lambda_2=0.05$. (b) $\Delta c_1=2.05, \lambda_2=0.01$. 

![Graph](image_url)

**Fig. 10.** Impact of $\lambda_2$ on profits in reactive strategies ($\Delta c_2=2, t=1$).
does lead to the reallocation of customers among the products. More customers who originally plan to buy the high-value product would turn to buy the low-value product when a backordering strategy is adopted. In contrast, a compensation strategy can keep more customers for the disrupted high-value product. The mixed strategy can keep more customers for the disrupted high-value product, and there is equal number of customers who would choose the downgraded version in both the downgrading strategy and the mixed strategy. The mixed strategy is the best reactive strategy and the backordering strategy is the worst one among the four reactive strategies for supply disruption of the high-value product.

This research provides critical values for helping managers and decision-makers choose the most robust reactive strategies in the presence of supply disruptions. In particular, the results apply to the situations where products are substitutable, and the customers’ demand is sensitive to price and delayed delivery. To our knowledge, this paper is among the first in supply disruption management to consider comparison of different reactive strategies.

It is necessary to point out a number of limitations of this research. First of all, we assume the manufacturer has a monopoly status and have no consideration for competition from other manufacturers. To consider competition, the demand model should be revised and a more complex demand model should be introduced. It is an interesting research direction to examine the reactive strategies with market competition. Secondly, we assume the duration of the supply disruption is stable and information is symmetric. In our model, when supply disruption occurs, the supplier would report the disruption event and notify the manufacturer the duration of the disruption. Future research should consider random duration and asymmetric information about supply disruption.

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Appendix A

Proof of Lemma 1. If \( \theta s_l - p_2 > \theta s_1 - p_1 \) and \( \theta s_2 - p_2 > 0 \), then a customer of type \( \theta \) would choose product B over A. The values of \( \theta \) that satisfy the conditions fall into the range

\[
\max \left\{ \frac{p_2 - p_1}{s_2 - s_1}, 1 \right\}.
\]

Customers whose type values fall into range

\[
\left. \frac{p_1}{s_2 - s_1} \max \left\{ \frac{p_2 - p_1}{s_2 - s_1} \right\} \right|_{s_2 > s_1} \]

would choose to buy product A. According to Assumption 1,

\[
\frac{p_2 - p_1}{s_2 - s_1} = \frac{p_2 s_2 - p_1 s_2}{s_2 - s_1} > 0. \]

Proof of Proposition 1. We can know that the Hessian matrix is negative semi-definite, which implies that \( \pi_\theta \) is joint concave in \( p_1 \) and \( p_2 \). Thus, there exists the optimal solution. Solving the first-order conditions, we can derive Proposition 1. \( \square \)

Proof of Proposition 2. According to the expected profit function, we can derive

\[
\frac{\partial^2 \pi_\theta}{\partial p_1^2} = \frac{2 \mu}{s_2 - s_1 - \Delta s_1} - \frac{2 \mu}{\Delta s_1 + \lambda s_1} < 0.
\]

Thus, \( \pi_\theta \) is concave in \( \Delta p_1 \), and there exists the optimal solution. Solving the first-order condition, we have Proposition 2. \( \square \)

Proof of Lemma 2. If \( \theta s_2 - p_2 > \theta (s_1 + \lambda s_1) - (p_1 + \Delta p_1) \) and \( \theta s_2 - p_2 > 0 \), then a customer of type \( \theta \) would choose product B over A. The values of \( \theta \) that satisfy the conditions fall into the range

\[
\max \left\{ \frac{p_2 - p_1}{s_2 - s_1 + \lambda s_1}, 1 \right\}.
\]

Customers whose type values fall into range

\[
\left. \frac{p_1}{s_2 - s_1 + \lambda s_1} \max \left\{ \frac{p_2 - p_1}{s_2 - s_1 + \lambda s_1} \right\} \right|_{s_2 > s_1 + \lambda s_1} \]

would choose delayed delivery of product A. According to Assumption 2,

\[
\frac{p_2 - p_1}{s_2 - s_1 + \lambda s_1} = \frac{p_2 (s_1 + \Delta s_1) - (p_1 + \Delta p_1) s_2}{s_2 (s_2 - s_1 + \lambda s_1)} > 0. \]

Proof of Lemma 3. If \( \theta s_2 - p_2 > \theta (s_1 + \Delta s_1) - (p_1 + \Delta p_1) \) and \( \theta s_2 - p_2 > 0 \), then a customer of type \( \theta \) would choose product B. The values of \( \theta \) that satisfy the conditions fall into the range

\[
\max \left\{ \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1}, 1 \right\}.
\]

According to Assumption 2

\[
\frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1} = \frac{p_2 (s_1 + \Delta s_1) - (p_1 + \Delta p_1) s_2}{s_2 (s_2 - s_1 - \Delta s_1)} > 0. \]

Thus,

\[
\max \left\{ \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1}, 1 \right\} = \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1}.
\]

If \( \theta (s_1 + \Delta s_1) - (p_1 + \Delta p_1) s_1 \) and \( \theta (s_1 + \Delta s_1) - (p_1 + \Delta p_1) > 0 \), then a customer of type \( \theta \) would choose to upgrade product A. The values of \( \theta \) that satisfy the conditions fall into the range

\[
\max \left\{ \frac{p_1 + \Delta p_1}{s_1 + \Delta s_1}, 1 \right\} = \frac{p_1 + \Delta p_1}{s_1 + \Delta s_1}.
\]

And those customers whose type values fall into range

\[
\left. \frac{p_1 + \Delta p_1}{s_1 + \Delta s_1} \max \left\{ \frac{p_1 + \Delta p_1}{s_1 + \Delta s_1}, 1 \right\} \right|_{s_1 + \Delta s_1} \]

would choose to wait for product A. According to Assumption 2,

\[
\frac{p_1 + \Delta p_1}{s_1 + \Delta s_1} = \frac{p_1 + \Delta p_1 (s_1 + \lambda s_1) - (p_1 + \Delta p_1) s_1}{(s_1 + \Delta s_1) (s_1 + \lambda s_1)} > 0. \]

Proof of Proposition 2. According to the expected profit function, we can derive

\[
\frac{\partial^2 \pi_\theta}{\partial p_1^2} = \frac{2 \mu}{s_2 - s_1 - \Delta s_1} - \frac{2 \mu}{\Delta s_1 + \lambda s_1} < 0.
\]

Thus, \( \pi_\theta \) is concave in \( \Delta p_1 \), and there exists the optimal solution. Solving the first-order condition, we have Proposition 2. \( \square \)
would choose to wait for the disrupted product A. According to Assumption 2,
\[ \frac{p_2 - p_1 + z_1}{s_2 - s_1 + z_1 I} \cdot \frac{p_2}{s_2} = \frac{p_2(s_1 - z_1 I) - (p_1 - z_1 I)s_2}{s_2(s_2 - s_1 + z_1 I)} > 0. \]  
\[ \square \]

**Proof of Proposition 3.** According to the expected profit function, we derive
\[ \frac{\partial^2 \Pi_e}{\partial z_1^2} = -\frac{2 \mu s_2^2}{(s_2 - s_1 + z_1 I)(s_1 - z_1 I)} < 0. \]
Thus, \( \Pi_e \) is concave in \( z_1 \), and there exists the optimal solution. Solving the first-order condition, we have Proposition 3.  
\[ \square \]

**Proof of Lemma 5.** If \( \theta s_2 - p_2 > \theta (s_1 + \Delta s_1) - (p_1 + \Delta p_1) \) and \( \theta s_2 - p_2 > 0 \), then a customer of type \( \theta \) would choose product B. The values of \( \theta \) that satisfy the conditions fall into the range
\[ \left[ \max \left\{ \frac{p_2}{s_2} - \frac{p_2 - p_1 - \Delta p_1}{s_1}, \frac{1}{s_2} \right\}, \frac{1}{s_1} \right]. \]

If \( \theta (s_1 + \Delta s_1) - (p_1 + \Delta p_1) > \theta (s_1 - z_1 I) - (p_1 - z_1 I) \) and \( \theta (s_1 + \Delta s_1) - (p_1 + \Delta p_1) > 0 \), then a customer of type \( \theta \) would choose to buy the upgraded version of product A. The values of \( \theta \) that satisfy the conditions fall into the range
\[ \left[ \max \left\{ \frac{p_1 + \Delta p_1 - \Delta p_1}{s_1} + z_1 I, \frac{1}{s_2} \right\}, \frac{1}{s_1} \right]. \]

Customers whose type values fall into range
\[ \left[ \frac{p_1 - z_1 I}{s_1}, \frac{p_1 + \Delta p_1}{s_1 + \Delta s_1} \right], \frac{1}{s_2} \]
would choose to order product A and be compensated for late delivery. According to Assumption 2,
\[ \frac{p_2 - p_1 - \Delta p_1}{s_2 - s_1 - \Delta s_1} \cdot \frac{p_2}{s_2} = \frac{p_2(s_1 + \Delta s_1) - (p_1 + \Delta p_1)s_2}{s_2(s_2 - s_1 - \Delta s_1)} > 0. \]  
\[ \square \]

**Proof of Proposition 4.** We can know that the Hessian matrix is negative semi-definite, which implies that \( \tau^2_{m n} \) is joint-concave in \( \Delta p_1 \) and \( z_1 \). Thus, there exist the optimal solutions. Solving the first-order conditions, we can derive Proposition 4.  
\[ \square \]

**Proof of Proposition 5.** We can compute,
\[ \tau^2_{s 1} = \frac{s_2 \mu(s_1 I)^2}{4(s_2 - s_1 + z_1 I)(s_1 - z_1 I)} > 0, \]
\[ \tau^2_{s 2} = \frac{\mu(s_2 - s_1 + z_1 I)^2}{4(s_2 - s_1 + z_1 I)(s_2 - s_1 + z_1 I)} > 0. \]  
\[ \square \]

**Proof of Proposition 6.** We can compute,
\[ \tau^2_{m n} = \frac{\mu(s_2 - s_1 + z_1 I)^2}{4(s_2 - s_1 + z_1 I)(s_2 - s_1 + z_1 I)} > 0, \]
\[ \tau^2_{m n} = \frac{s_2 \mu(s_1 I)^2}{4(s_2 - s_1 + z_1 I)(s_1 - z_1 I)} > 0. \]  
\[ \square \]

**Proof of Lemma 6.** If \( \theta (s_2 - z_2 I) - p_2 > \theta s_1 - p_1 \) and \( \theta (s_2 - z_2 I) - p_2 > 0 \), then a customer of type \( \theta \) would choose product B. The values of \( \theta \) that satisfy the conditions fall into the range
\[ \left[ \max \left\{ \frac{p_2 - p_1}{s_2 - s_1 - z_2 I}, \frac{p_2}{s_2 - z_2 I} \right\}, \frac{1}{s_2} \right]. \]

Customers whose type values fall into range
\[ \left[ \max \left\{ \frac{p_2 - p_1}{s_2 - z_2 I}, \frac{p_2}{s_2 - z_2 I} \right\}, \frac{1}{s_2} \right]. \]
would choose product A. According to Assumption 1,
\[ \frac{p_2 - p_1}{s_2 - s_1 - z_2 I} - \frac{p_2}{s_2 - z_2 I} = \frac{p_2 s_1 - p_2 s_1}{s_2 - s_1 - z_2 I} > 0. \]

If \( \theta (s_2 - z_2 I) - p_2 > \theta s_1 - p_1 \) and \( \theta (s_2 - z_2 I) - p_2 > 0 \), then a customer of type \( \theta \) would choose product B. The values of \( \theta \) that satisfy the conditions fall into the range
\[ \left[ \max \left\{ \frac{p_2 - p_1}{s_2 - z_2 I}, \frac{\Delta p_2}{s_2 - z_2 I} \right\}, \frac{1}{s_2} \right]. \]

According to Assumption 3,
\[ \frac{\Delta p_2}{s_2 - z_2 I} - \frac{p_2}{s_2 - z_2 I} = \frac{p_2 - \Delta p_2}{s_2 - z_2 I} > 0. \]
Thus

$$\max \left\{ \frac{p_2-z_2 l}{s_2-\Delta s_2^{l}}, \frac{\Delta p_2-z_2 l}{\Delta s_2^{l}, s_2-\Delta s_2^{l}} \right\}$$

If $0(s_2-\Delta s_2^{l})(p_2-\Delta p_2)>(s_1-p_1)1$ and $0(s_2-\Delta s_2^{l})(s_2-\Delta s_2^{l})>0$, then a customer of type $\theta$ would choose the downgraded version of product B. The values of $\theta$ that satisfy the conditions fall into the range

$$\max \left\{ \frac{p_2-\Delta p_2}{s_2-\Delta s_2^{l}}, \frac{p_2-\Delta p_2-p_1}{s_2-\Delta s_2^{l}}, \frac{\Delta p_2-z_2 l}{\Delta s_2^{l}, s_2-\Delta s_2^{l}} \right\}$$

And customers whose type values fall into range

$$\left\{ \frac{p_1, \max \left\{ p_2-\Delta p_2, p_2-\Delta p_2-p_1 \right\}}{s_2-\Delta s_2^{l}, s_2-\Delta s_2^{l}, s_2-\Delta s_2^{l}} \right\}$$

would choose to buy product A. Similarly, according to Assumption 3,

$$p_2-\Delta p_2-p_1 \frac{p_2-\Delta p_2}{s_2-\Delta s_2^{l}, s_2-\Delta s_2^{l}}=\frac{(p_2-\Delta p_2)s_1-p_1(s_2-\Delta s_2^{l})}{s_2-\Delta s_2^{l}, s_2-\Delta s_2^{l}}=0.$$  □

**Proof of Proposition 7.** According to Lemma 6, we can compute

$$n_{a}^{l} \cdot n_{a}^{l} = \frac{\mu(l)^{2}}{4(s_2-\Delta s_2^{l}, s_1)} > 0,$$

$$n_{a}^{l} \cdot n_{a}^{l} = \frac{\mu(l)(c_2-c_1)(s_2-\Delta s_2^{l}, s_1) \Delta s_2^{l}}{4(s_2-\Delta s_2^{l}, s_1)(s_2-\Delta s_2^{l}, s_1) \Delta s_2^{l}} > 0,$$

$$n_{a}^{l} \cdot n_{a}^{l} = \frac{\mu(l)(c_2-c_1)(s_2-\Delta s_2^{l}, s_1) \Delta s_2^{l}}{4(s_2-\Delta s_2^{l}, s_1)(s_2-\Delta s_2^{l}, s_1) \Delta s_2^{l}} > 0,$$

$$n_{a}^{l} \cdot n_{a}^{l} = \frac{\mu(l)^{2}}{4(s_2-\Delta s_2^{l}, s_1)} > 0.$$  □

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