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Supply chain coordination through cooperative advertising with reference price effect

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ABSTRACT

Cooperative advertising, which usually occurs in a vertical supply chain, is typically a cost sharing and promotion mechanism for the manufacturer to affect retail performance. Research in the literature, however, rarely considers the important phenomenon that advertising has a positive effect on the consumer's reference price. In fact, when a consumer makes a decision to buy a product or not, a reference price is usually in his mind and plays a determinant role. Taking into account the impact of advertising on the reference price, this paper proposes a dynamic cooperative advertising model for a manufacturer–retailer supply chain and analyzes how the reference price effect would influence the decisions of all the channel members. In our model, both the consumer's goodwill and reference price for the product are assumed to be influenced by the advertising and are modeled in differential dynamic equations. In addition, the advertising level, the consumer's goodwill and the reference price are all assumed to have positive effect on sales. Utilizing differential game theory, this paper formulates the optimal decisions of the manufacturer and the retailer in two different game scenarios: Stackelberg game and cooperative game. Also, this paper proposes a new mechanism to coordinate the supply chain in which both the manufacturer and the retailer share each other's advertising costs.

1. Introduction

Cooperative advertising, which usually occurred in a vertical supply chain, is typically a cost sharing and promotional mechanism for manufacturers to affect retailer performance. As distinguished by Huang and Li [18] and Huang et al. [19], advertising can be divided into national and local advertising. National advertising mainly focuses on influencing potential consumers to consider a particular brand and develops brand knowledge and preference, whereas local advertising intends to stimulate consumers' instant buying behavior. To encourage a retailer to invest more in local advertising, the manufacturer usually shares part of the retailer's local advertising cost.

As a typical issue of supply chain coordination, the cooperative advertising program has received significant attention in business and academics. As indicated by Nagler [35], the total expenditure on cooperative advertising in 2000 was estimated at $15 billion in the USA, nearly a four-fold increase in real terms in comparison to $900 million in 1970. Karray and Zaccour [27] also indicated that marketing research firms like National Register Publishing has collected more than 4000 co-op programs subsidized by manufacturers in 52 product classifications.

In 1973, Berger was the first to propose a primary cooperative advertising model [5]. After that, Dant and Berger [9], Huang and Li [18], Huang et al. [19], Li et al. [32], Jørgensen et al. [20,21], Yue et al. [51] and Xie and Neyret [49] extended different aspects of Berger's work. There are two types of modeling in co-op advertising: static and dynamic. For example, Huang and Li [18], Huang et al. [19] and Li et al. [32] used static models to extended Berger's work in a supply chain framework; Yue et al. [51] used a static model to study the cooperative advertising problem by considering price discount in demand elasticity market circumstance, and Xie and Neyret [49] used a static model to propose a more general model by including the cooperative advertising and pricing decisions simultaneously.

For the dynamic models, readers may refer to Chintagunta and Jain [7], Jørgensen et al. [20,21], and Karray and Zaccour [26]. Utilizing the Nerlove–Arrow framework, Chintagunta and Jain [7] developed a dynamic model to determine the channel member's equilibrium marketing efforts for a manufacturer–retailer supply chain. Jørgensen et al. [20] extended the work of Chintagunta and Jain [7] for cooperative advertising by considering that both channel members made long term and short term advertising efforts to enhance sales and consumer goodwill, whereas Jørgensen et al. [21] assumed decreasing marginal returns to

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goodwill and adopted a more flexible functional form for the sales dynamics. These works were extended by Karray and Zaccour [26], in which the retailer sold both his own private product and the manufacturer's product. Further, under the dynamic framework, He et al. [15] studied the cooperative advertising program for a supply chain system consisting of a single manufacturer and two competing retailers.

Although cooperative advertising has been extensively studied during the past several decades, an important marketing phenomenon, i.e., reference price effect, has not been considered in the cooperative advertising models. In fact, reference dependence has a long-standing tradition in psychology and has been the focus of a great deal of empirical research [4,6,30]. According to Kalwani et al. [23], when a consumer buys a product, he usually has a reference price r in his mind. If he finds the current price p is less than his reference price r, he will feel a sense of gain and the sales of the product would increase. On the contrary, if $r < p$, the consumer will feel a sense of loss and sales would decrease. In the literature, the impact of reference price on demand is called reference price effect [37]. As argued by Mazumdar et al. [34], reference price represents a consumer's evaluation of a product; the reference price is affected by many factors such as price (including the historical price, the suggested retail price, the rival product's price, etc.), advertisement, quantity, delays, and so on.

Due to its significant influence on consumers' behaviors, reference price has received much attention from researchers. Thaler [47] explained that the impact of a reference price on consumer demand was influenced by the dynamic comparison between reference price and current market price. Pulter [38] further indicated that an actual price which is higher or lower than the reference price has a different impact on demand. Greenleaf [12] showed that reference price effect could increase the impact of promotions. Taking this effect into account, Greenleaf demonstrated how a retailer should develop an optimal strategy for long-term promotions to maximize his profits. Kopalle et al. [29] considered customer heterogeneity in a study of reference price effect and showed that cyclical pricing policies were optimal. Taking threshold effects into account, Raman and Bass [40] provided a test of reference price theory and assumed a basic asymmetry in market response. Their research suggested that a price-promotion would not be noticed by consumers except if it exceeded a minimum threshold. Fibich et al. [14] studied how the profitability of price promotion was affected by the asymmetric reference price effect and results showed that if effects of loss on demand were larger than that of gain, price promotions could lead to a decline in profit, and vice versa.

Because reference price has a significant impact on consumer buying decisions and because reference price can be affected by advertising, it is necessary to consider the reference price effect in cooperative advertising models so that managers can make better decisions about promotional strategies. In this paper, we consider a dynamic cooperative advertising model with reference price effect for a manufacturer–retailer supply chain. In our model, both the consumer's goodwill and reference price for a product are assumed to be influenced by advertising and are modeled in differential dynamic equations. In addition, the advertising efforts, the consumer's goodwill, and the reference price are all assumed to have a positive effect on product sales. Utilizing differential game theory, we obtain the optimal advertising decisions of the manufacturer and retailer under two different game scenarios, i.e., the Stackelberg game and the cooperative game structure. Also, we propose a new cooperation mechanism to coordinate the supply chain in which both the manufacturer and retailer share the other's advertising costs.

Some interesting results of this paper include the following: (i) as opposed to most previous literature, which did not take into account advertising's impact on reference price and proved that the steady state reference price equals to the market price [13,37], our model illustrates that the steady state reference price is usually higher than the market price; (ii) due to the reference price effect, a firm will invest more in national advertising; the larger the impact of the reference price, the more national advertising should be invested in, and (iii) when the retailer has a relatively high profit, it is necessary for him to share a part of the manufacturer's national advertising costs, which is contrary to the common cooperative advertising practice by which the manufacturer usually shares the retailer's local advertising costs.

This paper is organized as follows. Literature reviews are in Section 2. In Section 3, the reference price effect is added into the dynamic cooperative advertising model. Based on the model, in Sections 4 and 5 we analyze the manufacturer and retailer's optimal decisions in the Stackelberg game and the cooperative game structure, respectively. A new supply chain coordination mechanism is introduced in Section 6. Concluding remarks are given in Section 7.

2. Literature review

Literature related to this paper focuses mainly on cooperative advertising and reference price effect. Research about cooperative advertising is usually divided into static models and dynamic models. For the static model, Berger [5] proposed a cooperative advertising model by taking the participation rate as the manufacturer's decision variable, i.e., the manufacturer was to decide the optimal cost sharing rate that he would undertake for the retailer. The model was then extended by Dant and Berger [9], who considered an uncertainty demand in franchising systems. Using game theory, Dant and Berger [9] obtained the manufacturer's optimal participation rate and the retailer's optimal advertising spending. Dividing the advertising into the retailer's local advertising and the manufacturer's national advertising, Huang and Li [18], Huang et al. [19], and Li et al. [32] further extended the manufacturer–retailer cooperative advertising model. Utilizing a game theory approach, the research discussed cooperative advertising problems under two scenarios, i.e., (i) the manufacturer was the leader and the retailer was the follower and (ii) the manufacturer and retailer made decisions in a co-op partnership.

For the dynamic model, Chintagunta and Jain [7] determined equilibrium marketing efforts for a manufacturer and a retailer. Then, Jørgensen et al. [20,21] examined the case where both channel members made long term and short term advertising efforts to enhance sales and consumer goodwill. Also, Jørgensen et al. [22] investigated an interesting phenomenon, i.e., the retailer's promotions could damage the brand image. With consideration of this phenomenon, the authors examined whether a cooperative program could still work in such a context. Then, Karray and Zaccour [26] extended the models of Jørgensen et al. [21] by considering a retailer who sold both his own products with a private label and the manufacturer's products while choosing promotion efforts for these two products. Furthermore, He et al. [16] modeled a typical supply chain problem as a stochastic Stackelberg differential game, in which the relationship between the manufacturer and retailer is asymmetric. Sigué and Chintagunta [45] studied a cooperative advertising problem by considering a franchise system consisted of a franchisor and two competing franchisees.

As an important factor that influences the market demand, the reference price has been researched since at least the 1980s. Lattin and Buckin [31] indicated that the reference price framework was consistent with several psychological theories of consumer behavior and price perception, including the adaptation-level theory [17] and the assimilation contrast theory [46]. Empirical research on the effects of reference price on consumer buying behavior usually utilizes two approaches. One is the household-level brand choice models which assume that the utility received by a consumer can be influenced by not only reference price, but also other factors such as brand image and household inventory level [42,48,24,31,23,36]. The other approach is to examine the effect of reference price on the market share of a brand [42,40,43,25].

Models of reference price effect mainly focus on pricing strategies. Fibich et al. [13] analyzed the asymmetric reference-price effects on the optimal pricing strategies in the infinite and finite planning horizon. To obtain explicit solutions to various non-smooth optimization pricing problems, Fibich et al. [13] used methods of open-loop and closed-loop equilibrium. Assuming that the demand was influenced by the market price, the reference price and the competitor's price simultaneously, Anderson et al. [3] proposed a temporal pricing model for a big-box retailer and a smaller local merchant with differing market powers. Popescu and Wu [37] considered the dynamic pricing problem of a monopolist firm in a market with repeated interactions, where the demand was sensitive to the firm's pricing history; results indicated that managers who ignored long-term implications of their pricing strategy (due to consumer memory and anchoring effects) would consistently price too low, thereby systematically losing revenue. Geng et al. [11] studied not only the retailer's pricing strategy with reference price effect, but also the manufacturer's wholesale price and the retailer's optimal promotion frequency decision. Their results showed that the retailer preferred a periodic promotion strategy to a constant price strategy only when the gain effect on demand was strictly greater than the loss effect.

Although some literature on pricing strategy has begun to consider the reference price effect, as far as we have been able to ascertain, there is no cooperative advertising literature that takes the reference price effect into account. Since advertising can improve the product's brand image and the consumer's reference price simultaneously, and the reference price has a significant impact on the consumer's buying behavior, it is necessary to study the reference price effect's impact on a cooperative advertising program. Therefore, we develop such a cooperative advertising model taking the reference price effect into consideration, and try to study its impact on cooperative advertising decisions.

3. Model development

The system considered in this paper consists of a manufacturer and a retailer. The manufacturer sells his product to the retailer while the retailer sells the product to the consumer. To improve sales, the manufacturer will usually invest in national advertising and the retailer in local advertising. We denote the manufacturer's national advertising level over time t as $a(t)$, and the retailer's local advertising level as $q(t)$.

Generally, when the two channel members advertise a product, a kind of goodwill will accumulate among the consumers. As with previous literature (e.g. [8,20,45]), we assume that the changing of the goodwill follows the Nerlove–Arrow framework, i.e.,

$$\dot{G}(t) = \theta_1 a + \theta_2 q - \delta G, \quad G(0) = G_0 \tag{1}$$

where $G(t)$ is the accumulated goodwill over time t, $G_0 > 0$ is the initial goodwill, θ_1, θ_2 are positive constants reflecting that the national and local advertising have positive effects on the accumulation of goodwill, and $\delta > 0$ is the diminishing rate of goodwill.

In addition, national and local advertising can also affect the consumer's reference price, the price that consumers maintain in their minds to be reasonable for a certain type of product. When consumers decide whether or not to buy a product, they will compare the product's price with their reference price. According to Mazumdar et al. [34], the reference price can be affected by the channel member's national and local advertising as well as by the consumer's memory of past prices. Denoting the consumer's reference price over time t as $r(t)$ and current price of the product is $p(t)$, we assume that changes in the product's reference price follow Eq. (2), i.e.,

$$\dot{r}(t) = \beta(p-r) + \mu_1 a + \mu_2 q, \quad r(0) = r_0 \tag{2}$$

where $r_0 = r(0)$ is the initial reference price, and β, μ_1 and μ_2 are all constants. In detail, the item $\beta(p-r)$ in Eq. (2) reflects the consumer's prior purchase experience, whereas the parameter $\beta > 0$, called "memory parameter" [13], characterizes the memory effect. A higher β implies the consumer has short term memory and less loyalty to the product. The two items $\mu_1 a$ and $\mu_2 q$ in Eq. (2) represent the effect of national and local advertising on the reference price. Generally, investments in national advertising can enhance a brand's image, which further increases the consumer's valuations on the product [41,10]. Thus $\mu_1 > 0$ and $\mu_2 > 0$ are assumed.

It should be noted that we will assume that $p(t)$ in Eq. (2) is kept as a constant later in our model due to the following reasons. First, the main focus of our paper is to consider the manufacturer and retailer's optimal advertising efforts when advertising can affect the consumer's reference price. Since our purpose is to disclose the impact of the reference price effect on the advertising decision making, we keep the retail price $p(t)$ fixed to avoid the price decision. Second, one may solve the optimal $p(t)$ under the framework of our paper. The optimal $p(t)$ will then be changed over time t, which means the retailer will change the retail price day to day. However, this is not true in most retail practice. As indicated by Lattin and Buckin [31], too much promotion and price discounting/changing might adversely affect brand choice behavior, e.g., a consumer exposed to frequent price promotions might become accustomed to finding the brand available on promotion at a discounted price, which in fact decreases the reference price. Therefore, neither the manufacturer nor the retailer would rather change the retailer price frequently. As an example, in the fashion apparel industry, the brand Hailanhome from Jiangsu of China would rather keep the retail price of its products fixed all the time. Third, once the price changes, the equilibrium advertising efforts can be recalculated by our model and thus the decisions can be easily updated.

Generally, the national and local advertising efforts, the consumer's goodwill and the reference price all have positive effects on the sales. Thus we assume the sales $S(t)$ to satisfy the following equation:

$$S(t) = \alpha(r-p) + G + \lambda_1 a + \lambda_2 q \tag{3}$$

where α, λ_1 and λ_2 are all positive constants. In Eq. (3), the item $\alpha(r-p)$ represents the reference price effects on sales. When a product's reference price r is larger than its current price p, the effect on sales is positive, whereas when $r < p$, the effect will be negative [39,33]. A higher α implies consumers are more sensitive to the gap between their reference price and the observed price. The item G illustrates the fact that sales will improve if the product has a higher goodwill among consumers, whereas the other two items $\lambda_1 a$ and $\lambda_2 q$ reflect that national and local advertising have positive instant effects on sales.

Similar to previous literature such as Jørgensen et al. [21], the advertising costs $C_i(i=M,R)$ are assumed quadratic, i.e.,

$$C_M=\tfrac{1}{2}a^2 \tag{4}$$

and

$$C_R=\tfrac{1}{2}q^2. \tag{5}$$

Without accounting for the advertising costs, assume that ρ_M and ρ_R are the marginal profits of the manufacturer and the retailer respectively. Furthermore, to stimulate the retailer to invest more in local advertising, the manufacturer will share a part of the retailer's advertising costs. Supposing that ϕ_1 is the participation rate that the manufacturer is willing to undertake for the retailer's advertising costs, the profit of manufacturer is then

$$\pi_M(t)=\rho_M S-\tfrac{1}{2}a^2-\tfrac{1}{2}\phi_1 q^2 \tag{6}$$

and that of the retailer is

$$\pi_R(t)=\rho_R S-\tfrac{1}{2}(1-\phi_1)q^2 \tag{7}$$

Thus, the profit of the whole system is

$$\pi(t)=(\rho_M+\rho_R)S-\tfrac{1}{2}a^2-\tfrac{1}{2}q^2 \tag{8}$$

We assume that participation rate ϕ_1 does not change over time due to several reasons. First, due to complexity issues, firms seldom provide cooperative advertising programs with a participation rate that changes over time. For example, Nagler [35] has made the following statement about the cooperative advertising programs in his empirical research database: "all the 1470 plans explicitly listed a single participation rate". If a firm provides a cooperative advertising program with a changing participation rate, the firm needs to know exactly his partner's daily advertising cost, which is much more difficult than knowing the entire advertising cost over a certain period of time. Second, even in the literature which models the participation rate as a function of time, the final optimal decision for participation rates were all constant over time (e.g. [20,22,26,45,16]). The main focus of our paper is not on the participation rate, but on the effect of the reference price on the cooperative advertising program. Consequently, we model the participation rate as a constant over time in order to reduce the mathematic difficulty.

The objectives of the manufacturer and the retailer are to achieve their optimal advertising efforts by maximizing the present value of their profits, i.e.,

$$\max_a J_M=\int_0^{+\infty} e^{-\rho t}\left[\rho_M S(t)-\tfrac{1}{2}a^2-\tfrac{1}{2}\phi_1 q^2\right]dt \tag{9}$$

and

$$\max_q J_R=\int_0^{+\infty} e^{-\rho t}\left[\rho_R S(t)-\tfrac{1}{2}(1-\phi_1)q^2\right]dt \tag{10}$$

where ρ is the discount rate. Further, when two channel members coordinate as a vertical integrated system, their objective is

$$\max_{a,q} J=\int_0^{+\infty} e^{-\rho t}\left[(\rho_M+\rho_R)S-\tfrac{1}{2}a^2-\tfrac{1}{2}q^2\right]dt \tag{11}$$

Since advertising has a decreasing marginal effect as advertising efforts increase, firms seldom increase their advertising levels infinitely. Thus, as with previous literature such as [44], we assume an upper bound exists for the control variables $a(t)$ and $q(t)$, i.e.,

$$0\le a(t),\quad q(t)\le M \tag{12}$$

where M is a large enough constant.

In the following sections, we will calculate the optimal advertising effort as well as the optimal proportion of local advertising costs paid by the manufacturer based on the Stackelberg game and a supply chain coordination mechanism, then analyze the relative coordination conditions in the coordination mechanism.

4. Stackelberg game (non-integrated decision)

When the two firms make decisions independently, the decision structure is assumed as follows. The manufacturer firstly offers the retailer a participation rate ϕ_1 that he is willing to assume for the local advertising costs. Once the participation rate is given, both the manufacturer and the retailer will decide their advertising efforts over time. Since such advertising decisions may change all the time, it is reasonable to assume that the two firms make their advertising efforts decisions simultaneously. This decision structure is a Stackelberg game. To solve this problem, we first calculate the equilibrium advertising efforts of the two firms for a specific given participation rate ϕ_1, and then we obtain the equilibrium present value of the two firms' profits. Since the present values are functions of the participation rate ϕ_1, we can then calculate the optimal ϕ_1 that maximizes the manufacturer's profit.

When the participation rate ϕ_1 is fixed, the manufacturer's objective is the equation given in Eq. (9) and the retailer's objective is the equation given in Eq. (10). Taking Eqs. (1) and (2) into account, the present value Hamiltonian for the manufacturer and the retailer are given as follows respectively, i.e.,

$$\begin{aligned} H_M=&\rho_M\left[\alpha(r-p)+G+\lambda_1 a+\lambda_2 q\right]-\tfrac{1}{2}a^2-\tfrac{1}{2}\phi_1 q^2\\ &+\gamma_{1M}(\beta(p-r)+\mu_1 a+\mu_2 q)+\gamma_{2M}(\theta_1 a+\theta_2 q-\delta G)\end{aligned} \tag{13}$$

and

$$\begin{aligned} H_R=&\rho_R\left[\alpha(r-p)+G+\lambda_1 a+\lambda_2 q\right]-\tfrac{1}{2}(1-\phi_1)q^2\\ &+\gamma_{1R}(\beta(p-r)+\mu_1 a+\mu_2 q)+\gamma_{2R}(\theta_1 a+\theta_2 q-\delta G)\end{aligned} \tag{14}$$

where γ_{1M}, γ_{2M} (γ_{1R}, γ_{2R}) represent the co-state variables in the manufacturer's (retailer's) problem associated with the changing of consumers' reference prices and goodwill levels, respectively.

From the present value Hamiltonian given by the above two equations, we calculate the equilibrium advertising efforts of the manufacturer and the retailer, and obtain Proposition 1.

Proposition 1. *When the manufacturer's participation rate ϕ_1 is given, the equilibrium advertising effort of the manufacturer is*

$$\overline{a}=\rho_M\lambda_1+\frac{\rho_M\theta_1}{\rho+\delta}+\frac{\rho_M\alpha\mu_1}{\rho+\beta} \tag{15}$$

and that of the retailer is

$$\overline{q}=\frac{1}{1-\phi_1}\left(\rho_R\lambda_2+\frac{\rho_R\theta_2}{\rho+\delta}+\frac{\rho_R\alpha\mu_2}{\rho+\beta}\right). \tag{16}$$

For the proof of each proposition, please see the Appendices. Proposition 1 illustrates the following results:

(i) Under the condition that $\phi_1=0$ in Eq. (16), we can see that the optimal local advertising level for the retailer has a similar structure to the manufacturer's national advertising effort given by Eq. (15). The three parts of Eq. (15) are $\rho_M\lambda_1$, $\rho_M\theta_1/(\rho+\delta)$ and $\rho_M\alpha\mu_1/(\rho+\beta)$. The first and the second parts are utilized to obtain the instant and long-term effect of the advertising to sales. The third part is the result of the consideration for the reference price effect. Noting that the third item will equal to zero if $\mu_1=0$, which implies that advertising has no impact on the reference price, the equilibrium advertising levels then just consist of the first two parts of the equation. Since both national and local advertising have usually a positive impact on the consumer's reference price, i.e., $\mu_1 > 0$ and $\mu_2 > 0$, it is shown that both the manufacturer and the retailer will increase the advertising level when they take the impact on the reference price into account. The more sensitive consumers are to the reference price, i.e., the larger α is, the more consciously the decision makers will consider this impact. On the other hand, if consumers have an extremely short memory or less loyalty to the product (i.e., a large β), the absolute value of the third part will reduce, and the consideration of the reference price may be ignored.

(ii) Differentiating the equilibrium advertising levels with their marginal profits respectively, we have $\partial\overline{a}/\partial\rho_M > 0, \partial\overline{q}/\partial\rho_R > 0$, $\partial\overline{a}/\partial\rho_R = 0$ and $\partial\overline{q}/\partial\rho_M > 0$. The first two equations imply that a higher marginal profit for the manufacturer and the retailer can stimulate them to invest more in their own advertising. The third equation illustrates that the retailer's marginal profit has no impact on the manufacturer's national advertising decision, whereas the fourth expression shows that the retailer's equilibrium level for local advertising is positively correlated to the manufacturer's marginal profit. In fact, if the manufacturer's marginal profit increases, his allowance policy for the retailer will increase too, and that will stimulate the retailer to invest more in local advertising.

Proposition 1. *Proves that the manufacturer's and the retailer's advertising efforts are both constants. Taking $\overline{a}$ and $\overline{q}$ into Eqs.* (9) and (10), *we get the reference price and the accumulated goodwill on products over time t as follows.*

Proposition 2. *Supposing that the national and local advertising efforts are kept as constants $\overline{a}$ and $\overline{q}$ respectively, the reference price and the accumulated goodwill on product over time t* are

$$r(t) = D_1 e^{-\beta t} + r_{SSS} \tag{17}$$

and

$$G(t) = D_2 e^{-\delta t} + G_{SSS} \tag{18}$$

where $D_1 = r_0 - r_{SSS}$, $r_{SSS} = p + (\mu_1\overline{a} + \mu_2\overline{q})/\beta$, $D_2 = G_0 - G_{SSS}$ and $G_{SSS} = (\theta_1\overline{a} + \theta_2\overline{q})/\delta$.

Note that the reference price and goodwill in Eqs. (17) and (18) will finally achieve their steady states r_{SSS} and G_{SSS} when $t \to +\infty$. The reference price's steady state, i.e., $r_{SSS} = p + (\mu_1\overline{a} + \mu_2\overline{q})/\beta$, is mainly affected by the product's market price and the advertising. When there is no advertising, the reference price will finally be the same as the market price. This conclusion was also reached by Fibich et al. [13]. However, when we take the advertising's impact into account, the steady reference price will usually increase, since both μ_1 and μ_2 are more likely positive. Additionally, the longer the consumers' memory of the product is, i.e., the smaller β is, the larger the steady reference price. Similarly, analyzing the expression of G_{SSS}, we find that the consumer's steady goodwill for the product is positively correlated to the national and local advertising efforts, and higher advertising efforts usually lead to a higher goodwill. In summary, advertising can not only accumulate a higher goodwill among consumers, but also increases the consumer's reference price.

Substituting Eqs. (17) and (18) into Eqs. (9) and (10) respectively, we get the manufacturer's and the retailer's present value of profit with fixed participation rate ϕ_1 as follows:

$$J_M = \frac{D_1\rho_M\alpha}{\rho+\beta} + \frac{\rho_M D_2}{\rho+\delta} + \frac{\alpha\rho_M(r_{SSS}-p)}{\rho} + \frac{\rho_M G_{SSS}}{\rho} + \frac{\rho_M\lambda_1\overline{a}}{\rho} + \frac{\rho_M\lambda_2\overline{q}}{\rho} - \frac{\overline{a}^2}{2\rho} - \frac{\phi_1\overline{q}^2}{2\rho} \tag{19}$$

and

$$J_R = \frac{D_1\rho_R\alpha}{\rho+\beta} + \frac{\rho_R D_2}{\rho+\delta} + \frac{\rho_R\alpha(r_{SSS}-p)}{\rho} + \frac{\rho_R G_{SSS}}{\rho} + \frac{\rho_R\lambda_1\overline{a}}{\rho} + \frac{\rho_R\lambda_2\overline{q}}{\rho} - \frac{(1-\phi_1)\overline{q}^2}{2\rho}. \tag{20}$$

Since the manufacturer gets the present value of his profit for any specific participation rate ϕ_1, he can then decide the optimal ϕ_1 to maximize the present value.

Proposition 3. *In the Stackelberg game, the manufacturer's optimal participation rate ϕ_1 is*

$$\overline{\phi}_1 = \begin{cases} \frac{2\rho_M-\rho_R}{2\rho_M+\rho_R} & \text{if } \frac{\rho_M}{\rho_R} \geq \frac{1}{2} \\ 0 & \text{else} \end{cases} \tag{21}$$

Differentiating the manufacturer's optimal participation rate $\overline{\phi}_1$ from the marginal profits of the two channel members when $\rho_M/\rho_R \geq 1/2$, we have $\partial\overline{\phi}_1/\partial\rho_M > 0$ and $\partial\overline{\phi}_1/\partial\rho_R < 0$. The two equations imply that higher marginal profit of the manufacturer or lower marginal profit of the retailer will lead to a larger local advertising allowance provided by the manufacturer. Generally, products with high marginal profit (e.g., appliances) are not frequently purchased by consumers and when people want to buy, they often make an overt search among local sources of information, seeking specific product information [18]. In order to give the retailer more incentive to attract consumers, the manufacturer should share more local advertising expenditures with the retailer. In practice, supermarkets and other large retailers, as compared to small retailers, usually have a lower retail price and a lower profit margin for a specific product. From Eq. (21) we determine that the manufacturer has a greater incentive to offer large retailers a higher advertising allowance. This outcome is in agreement with the results from other empirical research [53].

Substituting Eqs. (21) into (19) and (20) respectively, we can easily obtain the manufacturer and the retailer's optimal present values of their profits, i.e., $\overline{J}_M, \overline{J}_R$.

5. Coordination

In this section, we assume that the manufacturer and the retailer are vertically integrated and we calculate the system optimal decisions that maximize the present value of the whole supply chain profit. When the two firms coordinate as an integrated system, the system objective is given by Eq. (11). Taking the respective state Eqs. (1) and (2) into account, the present value Hamiltonian for the total channel is given as follows:

$$H = (\rho_M+\rho_R)[\alpha(r-p)+G+\lambda_1 a+\lambda_2 q] - \tfrac{1}{2}a^2 - \tfrac{1}{2}q^2 + \gamma_1(\beta(p-r)+\mu_1 a+\mu_2 q) + \gamma_2(\theta_1 a+\theta_2 q-\delta G) \tag{22}$$

where γ_1, γ_2 represent the co-state variables in the problem of total supply chain associated with the changing of consumers' reference prices and goodwill levels, respectively.

For the optimization problem above, the optimal advertising efforts of the two firms are proposed in Proposition 4.

Proposition 4. *When the manufacturer and the retailer coordinate as an integrated system, the optimal national and local advertising efforts over time t are both constants, i.e.,*

$$\tilde{a} = (\rho_M+\rho_R)\lambda_1 + \frac{(\rho_M+\rho_R)\theta_1}{\rho+\delta} + \frac{(\rho_M+\rho_R)\alpha\mu_1}{\rho+\beta} \tag{23}$$

and

$$\tilde{q} = (\rho_M+\rho_R)\lambda_2 + \frac{(\rho_M+\rho_R)\theta_2}{\rho+\delta} + \frac{(\rho_M+\rho_R)\alpha\mu_2}{\rho+\beta}. \tag{24}$$

Comparing the optimal advertising efforts with that of the Stackelberg game, it can be proven that both the national and local advertising efforts have been increased due to the coordination. Since it is obvious that the manufacturer's advertising efforts are higher after the coordination, we prove with the following equations that the retailer's advertising levels have also been improved. When $\rho_M/\rho_R \geq 1/2$, according to Eq. (21), we have $\overline{\phi}_1 = (2\rho_M - \rho_R)/(2\rho_M + \rho_R)$, then Eq. (16) can be transferred as follows, i.e.,

$$\overline{q} = \left(\rho_M + \tfrac{1}{2}\rho_R\right)\left(\lambda_2 + \frac{\theta_2}{\rho+\delta} + \frac{\alpha\mu_2}{\rho+\beta}\right). \tag{25}$$

Otherwise, when $0 \leq \rho_M/\rho_R < 1/2$, we have $\overline{\phi}_1 = 0$ and then Eq. (16) can be rewritten as

$$\overline{q} = \rho_R\left(\lambda_2 + \frac{\theta_2}{\rho+\delta} + \frac{\alpha\mu_2}{\rho+\beta}\right). \tag{26}$$

Comparing Eqs. (25) or (26) with the optimal local advertising levels given by Eq. (24), it is clear that the latter are larger.

Note that Proposition 4 also proves that the manufacturer and retailer's advertising efforts, i.e., $\tilde{a}$ and $\tilde{q}$, are both constants. Taking $\tilde{a}$ and $\tilde{q}$ into Eqs. (1) and (2), we get the reference price and the accumulated goodwill on products over time t as follows.

Proposition 5. *For the integrated system, supposing that both the national and local advertising efforts are kept fixed as constants $\tilde{a}$ and $\tilde{q}$ respectively, the reference price and the accumulated goodwill on products over time t are*

$$r(t) = E_1 e^{-\beta t} + r_{CSS} \tag{27}$$

and

$$G(t) = E_2 e^{-\delta t} + G_{CSS} \tag{28}$$

where $E_1 = r_0 - r_{CSS}$, $r_{CSS} = p + (\mu_1\tilde{a} + \mu_2\tilde{q})/\beta$, $E_2 = G_0 - G_{CSS}$, $G_{CSS} = (\theta_1\tilde{a} + \theta_2\tilde{q})/\delta$.

Comparing the above results with that of Proposition 2, we find that both the steady reference price and the steady goodwill are higher when the two channel members coordinate as an integrated system since both the national and local advertising efforts increase.

Substituting Eqs. (27) and (28) into (11), we get the present value of the total channel profit as follows:

$$\begin{aligned}\tilde{J} = {} & \frac{E_1(\rho_M+\rho_R)\alpha}{\rho+\beta} + \frac{(\rho_M+\rho_R)E_2}{\rho+\delta} \\ & + \frac{\alpha(\rho_M+\rho_R)(r_{CSS}-p)}{\rho} + \frac{(\rho_M+\rho_R)G_{CSS}}{\rho} \\ & + \frac{(\rho_M+\rho_R)\lambda_1\tilde{a}}{\rho} + \frac{(\rho_M+\rho_R)\lambda_2\tilde{q}}{\rho} - \frac{\tilde{a}^2}{2\rho} - \frac{\tilde{q}^2}{2\rho}\end{aligned} \tag{29}$$

After getting the present value of the integrated system profit in Eq. (29), we compare it with that of the Stackelberg situation. Changing α from 0 to 1 while keeping other parameters fixed, we calculate the difference between the two decision structures and draw the tendency in Fig. 1.

As shown in Fig. 1, Δa, Δq, ΔJ represent the differences between the cooperative and Stackelberg game structure for national advertising, local advertising and the total profit of the supply chain, respectively. When the two firms coordinate as an integrated system, both their national and local advertising levels will increase, and the system profit will also improve. Furthermore, we find that national advertising changes more sharply than local advertising. Noting that parameter α is utilized to reflect the degree of the reference price's impact on the consumer's buying behavior, the above fact implies that the larger the impact of the reference price, the more the system should or will invest in national advertising. In general, national advertising plays an important role in building a famous brand image and forming a higher reference price; consequently, national advertising is significant higher for products such as goods used daily and leisure foods whose reference prices both are easy to get and greatly affect the consumer's behavior.

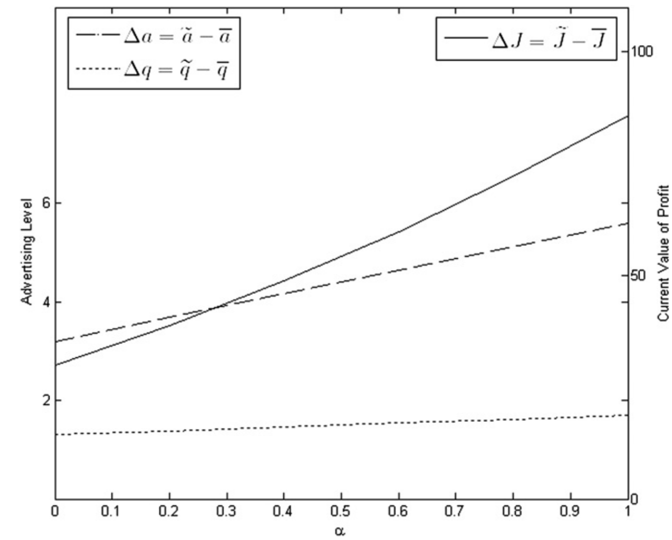

Fig. 1. Comparison between Stackelberg game and cooperative game.

6. A two-way subsidy policy

In Section 5, we reached the following conclusion: when the manufacturer and retailer coordinate as an integrated system, the optimal national and local advertising efforts are larger than in the Stackelberg game structure. Since the supply chain profits are usually higher in the cooperative game structure, it is necessary to design a contract so that the two channel members will choose the system optimal decision even when they make their decisions independently. To achieve this purpose, we introduce a two-way subsidy policy. Also, we will prove that the new contract can coordinate the supply chain smoothly.

The two-way subsidy policy works as follows: not only does the manufacturer share a part of the retailer's local advertising costs with a participant rate ϕ_1, but also the retailer shares a part of the manufacturer's national advertising costs with a participation rate of ϕ_2. Under the two-way subsidy policy, the profits of the manufacturer and retailer in Eqs. (6) and (7) are then changed into

$$\pi_M(t) = \rho_M S - \tfrac{1}{2}(1-\phi_2)a^2 - \tfrac{1}{2}\phi_1 q^2 \tag{30}$$

and

$$\pi_R(t) = \rho_R S - \tfrac{1}{2}\phi_2 a^2 - \tfrac{1}{2}(1-\phi_1)q^2 \tag{31}$$

whereas the present value Hamiltonian for the two members are

$$\begin{aligned}H_M = {} & \rho_M[\alpha(r-p) + G + \lambda_1 a + \lambda_2 q] - \tfrac{1}{2}(1-\phi_2)a^2 - \tfrac{1}{2}\phi_1 q^2 \\ & + \gamma_{1M}(\beta(p-r) + \mu_1 a + \mu_2 q) + \gamma_{2M}(\theta_1 a + \theta_2 q - \delta G)\end{aligned} \tag{32}$$

and

$$\begin{aligned}H_R = {} & \rho_R[\alpha(r-p) + G + \lambda_1 a + \lambda_2 q] - \tfrac{1}{2}\phi_2 a^2 - \tfrac{1}{2}(1-\phi_1)q^2 \\ & + \gamma_{1R}(\beta(p-r) + \mu_1 a + \mu_2 q) + \gamma_{2R}(\theta_1 a + \theta_2 q - \delta G).\end{aligned} \tag{33}$$

When the participation rates ϕ_1 and ϕ_2 are fixed, we obtain the equilibrium advertising efforts of the manufacturer and the retailer as follows.

Proposition 6. *When the two channel members work under the two-way subsidy policy as proposed above, supposing that the* *participation rates ϕ_1 and ϕ_2 are fixed, the manufacturer's equilibrium national advertising effort and the retailer's equilibrium local advertising effort are both constants, i.e.,*

$$\hat{a} = \frac{1}{1-\phi_2}\left(\lambda_1\rho_M + \frac{\theta_1\rho_M}{\rho+\delta} + \frac{\alpha\mu_1\rho_M}{\rho+\beta}\right) \tag{34}$$

and

$$\hat{q} = \frac{1}{1-\phi_1}\left(\lambda_2\rho_R + \frac{\theta_2\rho_R}{\rho+\delta} + \frac{\alpha\mu_2\rho_R}{\rho+\beta}\right). \tag{35}$$

Once there exists a specific value of ϕ_1 and ϕ_2 that can lead to the equilibrium advertising efforts given by Eqs. (34) and (35) equaling to those given by Eqs. (23) and (24) respectively, the manufacturer and retailer can be coordinated. Letting $\hat{a}=\tilde{a}$ and $\hat{q}=\tilde{q}$, and solving the two equations, we get the following proposition.

Proposition 7. *When the participation rates ϕ_1 and ϕ_2 of the two firms take the following values:*

$$\hat{\phi}_1 = \frac{\rho_M}{\rho_M+\rho_R} \tag{36}$$

and

$$\hat{\phi}_2 = \frac{\rho_R}{\rho_M+\rho_R}, \tag{37}$$

the supply chain system can be coordinated. That is to say the equilibrium effort for the national and local advertising is the same as that of the integrated system.

Comparing the participation rate $\hat{\phi}_1$ of the manufacturer in Eq. (36) with the manufacturer's optimal participation rate $\overline{\phi}_1$ in the Stackelberg game given by Eq. (21), we always have $\hat{\phi}_1 > \overline{\phi}_1$. Further, we also always have $\hat{\phi}_2 > 0$. These two facts mean that higher participation rates are needed to coordinate the supply chain under the two-way subsidy policy.

Once the supply chain has been coordinated, the reference price and the accumulated goodwill over time *t* are all the same as those given by Proposition 4. Substituting Eqs. (27) and (28) into Eqs. (30) and (31) and calculating their present values, we get the profit's present values for the manufacturer and the retailer under the proposed two-way subsidy policy as follows:

$$\hat{J}_M = \frac{F_1\rho_M\alpha}{\rho+\beta} + \frac{\rho_M F_2}{\rho+\delta} + \frac{\alpha\rho_M(r_{LSS}-p)}{\rho} + \frac{\rho_M G_{LSS}}{\rho} + \frac{\rho_M\lambda_1\hat{a}}{\rho} + \frac{\rho_M\lambda_2\hat{q}}{\rho} - \frac{(1-\hat{\phi}_2)\hat{a}^2}{2\rho} - \frac{\hat{\phi}_1\hat{q}^2}{2\rho} \tag{38}$$

and

$$\hat{J}_R = \frac{F_1\rho_R\alpha}{\rho+\beta} + \frac{\rho_R F_2}{\rho+\delta} + \frac{\alpha\rho_R(r_{LSS}-p)}{\rho} + \frac{\rho_R G_{LSS}}{\rho} + \frac{\rho_R\lambda_1\hat{a}}{\rho} + \frac{\rho_R\lambda_2\hat{q}}{\rho} - \frac{\hat{\phi}_2\hat{a}^2}{2\rho} - \frac{(1-\hat{\phi}_1)\hat{q}^2}{2\rho}. \tag{39}$$

Noting that conditions proposed by the following two equations may not always hold, i.e.,

$$\Delta J_M = \hat{J}_M - \bar{J}_M > 0 \tag{40}$$

and

$$\Delta J_R = \hat{J}_R - \bar{J}_R > 0 \tag{41}$$

the implication is that one of the two channel members may have a loss in profit, although the total profit for the system has improved. To illustrate this fact, we draw the tendency of ΔJ_M, ΔJ_R and $\Delta J = \tilde{J} - \bar{J}$ in Fig. 2. In Fig. 2, the value of the *x*-axis κ is defined as $\kappa = \rho_M/(\rho_M+\rho_R)$, which represents the manufacturer's marginal profit rate in the total supply chain. A larger κ means the manufacturer occupies more of the supply chain profit.

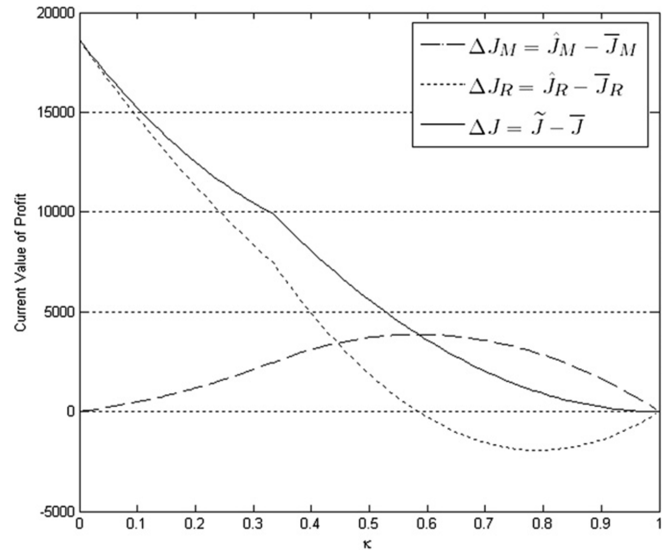

Fig. 2. Profit comparison between Stackelberg game and cooperative game.

As shown in Fig. 2, under the proposed two-way subsidy policy, the manufacturer can always have an improvement in profit, but the retailer's benefit may suffer when his profit margin is lower (i.e., κ is close to 1). Since the retailer's profit may be lower, he may not be willing to sign such a contract. However, this problem can be solved by a fixed transfer payment *b*. For instance, if there exists such a fixed transfer payment $b=(\Delta J_M - \Delta J_R)/2$, both the manufacturer and retailer can obtain an extra benefit from the coordination at a degree of $(\Delta J_M+\Delta J_R)/2$. Further, noting that there is a significant profit improvement when κ is small, it is reasonable for the retailer to provide a subsidy policy to the manufacturer for the national advertising if the retailer occupies most of the supply chain profit.

7. Concluding remarks

As an important consideration when consumers decide whether to buy a product or not, reference price has a significant impact on consumer behavior. Considering that national and local advertising can also influence the consumer's reference price, this paper investigates the cooperative advertising problem by taking reference price effect into account. Utilizing the differential game theory, the manufacturer and retailer's optimal efforts on national and local advertising are calculated in a Stackelberg game and a cooperative game structure. Also, a two-way subsidy policy has been proposed to coordinate the supply chain to allow both the manufacturer and retailer to share the other's advertising costs.

The main results of this paper include the following: (i) since advertising efforts have an impact on the consumer's reference price, the steady reference price will usually be higher than the market price, which is quite different from the conclusions in previous literature that usually proved that the steady reference price equals to the market price [13,37]; (ii) when the reference price has a large impact on consumer behavior, the firm should invest more in national advertising since it is more efficient in affecting the consumer's reference price; (iii) different from the previous cooperation mode in which the manufacturer usually shares a part of the retailer's local advertising costs [5,18], we find that it is necessary for a retailer to share a part of the manufacturer's national advertising costs if the retailer occupies most of the supply chain profit.

Some valuable extensions of this paper include the following. Firstly, the system that consists of a manufacturer and a retailer can be expanded to include multiple manufacturers or multiple retailers to take some interesting phenomena into consideration, including the symmetrical and asymmetrical substitution between products [28], and the price and advertising competition among channel members [54]. Secondly, it would be interesting to study a similar problem for hi-tech products with rapid technological innovation, under the assumption that a constant market price will not hold [50], while the new products' diffusion effect [2] should be well captured by models. Thirdly, the study can look at interesting consumer phenomena such as consumer inertia [52] and reference group effect [1] on cooperative advertising models. Finally, we assume that advertising can affect both consumer goodwill and the reference price, but whether there exist some relationships between them is not considered since there is no empirical study proving their relationship. Therefore, our model may be extended to consider the advertising effect once such a relationship has been discovered.

Acknowledgments

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Appendix A. Proof of Proposition 1

The necessary conditions for equilibrium are given by

$$\frac{\partial H_M}{\partial a}=0 \tag{A.1}$$

$$\frac{\partial H_M}{\partial \gamma_{1M}}=\dot{r} \tag{A.2}$$

$$\frac{\partial H_M}{\partial \gamma_{2M}}=\dot{G} \tag{A.3}$$

$$\dot{\gamma}_{1M}=\rho\gamma_{1M}-\frac{\partial H_M}{\partial r} \tag{A.4}$$

$$\dot{\gamma}_{2M}=\rho\gamma_{2M}-\frac{\partial H_M}{\partial G} \tag{A.5}$$

Eq. (A.1) implies

$$a=\rho_M\lambda_1+\mu_1\gamma_{1M}+\theta_1\gamma_{2M} \text{ or } \mu_1\gamma_{1M}=a-\rho_M\lambda_1-\theta_1\gamma_{2M} \tag{A.6}$$

Solving Eqs. (A.4) and (A.5), respectively, we get

$$\dot{\gamma}_{1M}=(\rho+\beta)\gamma_{1M}-\rho_M\alpha \tag{A.7}$$

and

$$\dot{\gamma}_{2M}=(\rho+\delta)\gamma_{2M}-\rho_M \tag{A.8}$$

Differentiating (A.6) with respect to time and substituting for the time derivative of γ_{1M} and γ_{2M} in Eqs. (A.7) and (A.8), we get

$$\dot{a}=(\rho+\beta)\mu_1\gamma_{1M}+(\rho+\delta)\theta_1\gamma_{2M}-\rho_M\alpha\mu_1-\rho_M\theta_1 \tag{A.9}$$

Substituting for $\mu_1\gamma_{1M}$ in (A.9) by Eq. (A.6), we get

$$\dot{a}=(\rho+\beta)a+(\delta-\beta)\theta_1\gamma_{2M}-(\rho+\beta)\rho_M\lambda_1-\rho_M\alpha\mu_1-\rho_M\theta_1 \tag{A.10}$$

or

$$(\delta-\beta)\theta_1\gamma_{2M}=\dot{a}-(\rho+\beta)a+(\rho+\beta)\rho_M\lambda_1+\rho_M\alpha\mu_1+\rho_M\theta_1 \tag{A.11}$$

Differentiating (A.10) with respect to time and substituting for γ_{1M},γ_{2M} in Eqs. (A.6) and (A.11), respectively, we get

$$\begin{aligned}\ddot{a}&=(2\rho+\beta+\delta)\dot{a}-(\rho+\delta)(\rho+\beta)a\\&\quad+(\rho+\delta)(\rho+\beta)\rho_M\lambda_1+(\rho+\delta)\rho_M\alpha\mu_1+(\rho+\beta)\rho_M\theta_1\end{aligned} \tag{A.12}$$

Similarly, considering the retailer's profit maximizing problem, we obtain

$$\begin{aligned}(1-\phi_1)\ddot{q}&=(2\rho+\beta+\delta)(1-\phi_1)\dot{q}-(\rho+\delta)(\rho+\beta)(1-\phi_1)q\\&\quad+(\rho+\delta)(\rho+\beta)\rho_R\lambda_2+(\rho+\delta)\rho_R\alpha\mu_2+(\rho+\beta)\rho_R\theta_2\end{aligned} \tag{A.13}$$

Solving Eqs. (A.12) and (A.13) to get the time paths of a and q, we get

$$a(t)=C_1e^{(\rho+\delta)t}+C_2e^{(\rho+\beta)t}+\overline{a} \tag{A.14}$$

and

$$q(t)=C_3e^{(1-\phi_1)(\rho+\delta)t}+C_4e^{(1-\phi_1)(\rho+\beta)t}+\overline{q} \tag{A.15}$$

where $\overline{a}=\rho_M\lambda_1+\rho_M\theta_1/\rho+\delta+\rho_M\alpha\mu_1/\rho+\beta$, $\overline{q}=1/1-\phi_1(\rho_R\lambda_2+\rho_R\theta_2/\rho+\delta+\rho_R\alpha\mu_2/\rho+\beta)$, C_1, C_2, C_3 and C_4 are parameters to be determined. Once $C_i\neq 0(i=1,\ldots,4)$, we know that $a(t)$ and $q(t)$ given by Eqs. (A.14) and (A.15) will be infinite when $t\to+\infty$, which do not satisfy the constraint given by Eq. (12). Thus, we have $C_i=0(i=1,\ldots,4)$, and the equilibrium $a(t)$ and $q(t)$ are constant, so that

$$\overline{a}=\rho_M\lambda_1+\frac{\rho_M\theta_1}{\rho+\delta}+\frac{\rho_M\alpha\mu_1}{\rho+\beta} \tag{A.16}$$

and

$$\overline{q}=\frac{1}{1-\phi_1}\left(\rho_R\lambda_2+\frac{\rho_R\theta_2}{\rho+\delta}+\frac{\rho_R\alpha\mu_2}{\rho+\beta}\right) \tag{A.17}$$

Proofs for Propositions 4 and 6 are similar to Appendix A and we ignore them in the following appendix.

Appendix B. Proof of Proposition 2

Substituting $a(t)=\overline{a}$ and $q(t)=\overline{q}$ into Eqs. (1) and (2), respectively, we have

$$\frac{dG(t)}{dt}=\theta_1\overline{a}+\theta_2\overline{q}-\delta G(t) \tag{B.1}$$

and

$$\frac{dr(t)}{dt}=\beta(p-r)+\mu_1\overline{a}+\mu_2\overline{q} \tag{B.2}$$

The general solutions of Eqs. (B.1) and (B.2) are

$$G(t)=D_2e^{-\delta t}+G_{SSS} \tag{B.3}$$

and

$$r(t)=D_1e^{-\beta t}+r_{SSS} \tag{B.4}$$

where $G_{SSS}=(\theta_1\overline{a}+\theta_2\overline{q})/\delta$, $r_{SSS}=p+(\mu_1\overline{a}+\mu_2\overline{q})/\beta$

D_1, D_2 are both arbitrary constants. Let $t=0$ in Eqs. (B.3) and (B.4), utilizing the initial conditions of Eqs. (1) and (2), we get $D_1=r_0-r_{SSS}$, $D_2=G_0-G_{SSS}$.

The proof for Proposition 5 is similar to Appendix B and we ignore it in the following appendix.

Appendix C. Proof of Proposition 3

Letting

$$\frac{d\bar{J}_M}{d\phi_1}=0 \tag{C.1}$$

we have

$$\phi_1=\frac{2\rho_M-\rho_R}{2\rho_M+\rho_R} \tag{C.2}$$

Considering that $\phi_1\in[0,1]$ we have

$$\phi_1=\begin{cases}\frac{2\rho_M-\rho_R}{2\rho_M+\rho_R} & \text{if } \frac{\rho_M}{\rho_R}\geq\frac{1}{2}\\ 0 & \text{else}\end{cases} \tag{C.3}$$

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