

Deflection coefficient maximization criterion based optimal cooperative spectrum sensing

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Received 22 January 2009; accepted 17 June 2009

Abstract

This paper proposes an optimal cooperative spectrum sensing scheme, based on the criterion of deflection coefficient maximization of the global decision statistic. Multiple cooperative secondary users serve in the cognitive radio network to provide space diversity for spectrum sensing. After the fusion center acquires the optimal fusion weights, an optimal global threshold setting strategy is utilized to obtain the final global decision. Since the proposed optimal cooperative sensing scheme requires precise estimations of primary user signal strengths and the noise variances at different cooperative secondary users, a recursive estimate algorithm is also proposed. Simulations illustrate the proposed optimal soft fusion scheme can significantly improve the spectrum sensing performance and outperform the conventional maximal-ratio combining and equal gain combining schemes. The recursive estimate algorithm can effectively approach the ideal performance of the proposed sensing scheme. © 2009 Elsevier GmbH. All rights reserved.

Keywords: Deflection coefficient maximization; Energy detection; Optimal soft fusion; Cooperative spectrum sensing; Cognitive radio

1. Introduction

Cognitive radio (CR) has been proposed in recent years as a promising paradigm for exploiting the precious spectrum opportunities, which are wasted by the current fixed spectrum allocation scheme, to solve the spectrum scarcity problem in nowadays [1,2]. Being inherently lower priority or secondary users (SU), the fundamental requirement for CR is to avoid interference to the primary users (PU) in the vicinity. In order to detect the PU signal with unknown location, structure and strength, energy detection (ED) serves as the optimal spectrum sensing scheme when the detector only knows the power of the received signal. Moreover, ED is also the most commonly used strategy in spectrum sensing due to its implementation simplicity [3,4].

However, there are several factors that prevent the energy detector from operating in a reliable manner, such as multipath fading/shadowing and noise power fluctuating [5,6]. These factors suggest the necessity of secondary users' cooperation in the CR networks [7–12].

In a centralized user cooperation scenario, several deflection coefficient (DC) based soft fusion algorithms have been proposed in the literature to improve the overall sensing performance in the CR network. In [9], a soft fusion solution is obtained by tackling the problem of maximizing the modified deflection coefficient (MDC). Based on the Rayleigh–Ritz inequality, this solution is actually a maximum eigenvector based soft fusion scheme. In [10], a linear-quadratic fusion strategy is proposed, on the basis of deflection criterion, to study its performance in correlated log-normal shadowing environments. Therein, the conventional ED sensing scheme has been rebuilt by introducing the covariance matrix of the received signal into the sensing statistic. In [11], an optimal soft fusion weight vector is derived in a likelihood ratio test. It is thereby proved to be a

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conventional maximal ratio combination (MRC) scheme. In [12], a blindly combined energy detection (BCED) for spectrum sensing is proposed. The blind combining weights are derived based on the covariance matrix of the signal samples received by multiple antennas of the fusion center (FC). Assuming the signal-to-noise ratios (SNR) of the received PU signals at the cooperative SUs are known, the proposed scheme in [9] is a special case of the BCED.

In this paper, an ED based optimal cooperative spectrum sensing scheme is developed, based on the optimality criterion of deflection coefficient maximization (DCM) of the global test statistic at the FC. The DCM based cooperative spectrum sensing scheme is categorized into the normal DCM (NDCM) soft fusion and the modified DCM (MDCM) one. We prove that the NDCM and MDCM based soft fusions have the same theoretical performance in cooperative spectrum sensing. To implement the proposed optimal sensing scheme, an optimal global threshold setting method and a simple yet effective recursive estimate algorithm are also proposed. Using this estimate algorithm, NDCM and MDCM only have trivial performance difference in practice. Analysis and simulations verify the superior performance of our proposed cooperative spectrum sensing scheme, compared to the conventional maximal-ratio combining (MRC) and equal gain combining (EGC) schemes.

The rest of the paper is organized as follows. In Section 2, the system model of spectrum sensing observation relaying and energy measuring is given. In Section 3, the DCM based cooperative spectrum sensing is investigated with two different fusion methods, namely the NDCM and MDCM. Implementation of the DCM based cooperative spectrum sensing scheme is then presented in Section 4. Conclusions are finally given in Section 5.

2. System model

2.1. Spectrum sensing observation relaying

We consider that M cooperative SUs (denoted as $\{R_i\}_{i=1}^M$) are deployed over a certain geographical area of the CR network by some upper layer algorithms and they simply serve as relays in the network to provide space diversity.

In the first phase, the signal received at the i -th relay R_i is

$$x_i(k) = \begin{cases} n_i(k), & \mathcal{H}_0, k \in \{1, 2, \dots, K\}, \\ \sqrt{E_{PU}}h_i s(k) + n_i(k), & \mathcal{H}_1, i \in \{1, 2, \dots, M\}, \end{cases} \quad (1)$$

where $s(k)$ is the transmitted signal of the PU transmitter at time k with unit power, $\sqrt{E_{PU}}$ is the amplitude of the PU signal yielding a transmitting power of E_{PU} , and h_i is the channel gain between the PU and R_i , which accommodates the effects of channel shadowing, channel loss and fading, etc. $n_i(k)$ is the complex additive white Gaussian

noise (AWGN) with zero mean and variance σ_i^2 , and it is assumed that $n_i(k)$ and $s(k)$ are mutually independent. \mathcal{H}_0 and \mathcal{H}_1 are the hypotheses of the PU signal being absent and present, respectively. K is the time-bandwidth product $2T_S W$, where T_S is the effective spectrum sensing interval and W is the bandwidth of the licensed spectrum of interest.

Upon receiving signals in the first phase, each relay will simply acts in an amplify-and-forward (AAF) manner, and the signal received by the FC from R_i is

$$y_i(k) = \sqrt{E_i}\bar{h}_i x_i(k) + n_{FC}(k), \\ = \begin{cases} \sqrt{E_i}\bar{h}_i n_i(k) + n_{FC}(k), & \mathcal{H}_0, \\ \sqrt{E_{PU}E_i}\bar{h}_i h_i s(k) + \tilde{n}_{FC,i}(k), & \mathcal{H}_1, \end{cases} \quad (2)$$

where \bar{h}_i is the channel gain between the FC and R_i , $n_{FC}(k)$ is the complex AWGN noise at the FC with zero mean and variance σ_{FC}^2 , and E_i is the transmit power of R_i . Here, E_i has two physical meanings. First, it represents the supplied power that R_i can provide, for instance, in a battery-supported scenario; second, it functions as an adjustable parameter that can be controlled by the FC to optimize the power allocation in the CR network [13].

As for the noise and signal components at the FC, again, it is assumed that:

- (1) $n_{FC}(k)$ is independent with both $n_i(k)$ and $s(k)$;
- (2) $n_{FC}(k)$ is statistically the same for each of the relays;
- (3) the individual sensing observations are relayed to the FC in a space orthogonal manner that the FC can easily discern the M observations captured at different R_i .

Consequently, the equivalent noise variance of $\tilde{n}_{FC,i}(k)$ is

$$\tilde{\sigma}_{FC,i}^2 = E_i |\bar{h}_i|^2 \sigma_i^2 + \sigma_{FC}^2, \quad i \in \{1, 2, \dots, M\}. \quad (3)$$

We can now write the received signals by the FC at time k in a more compact form

$$\mathbf{y}(k) = \begin{cases} \mathbf{\Pi}_0 \times \mathbf{n}(k) + n_{FC}(k)\mathbf{1}, & \mathcal{H}_0, \\ \mathbf{\Pi}_1 \times s(k)\mathbf{1} + \tilde{\mathbf{n}}_{FC}(k), & \mathcal{H}_1, \end{cases} \quad (4)$$

where the signals $\mathbf{y}(k) = [y_1(k), y_2(k), \dots, y_M(k)]^T$ are received by the FC, $\mathbf{n}(k) = [n_1(k), n_2(k), \dots, n_M(k)]^T$ are the noise components at the M cooperative relays, $\tilde{\mathbf{n}}_{FC}(k) = [\tilde{n}_{FC,1}(k), \tilde{n}_{FC,2}(k), \dots, \tilde{n}_{FC,M}(k)]^T$ are the equivalently combined M noise components at the FC, and $\mathbf{1}$ is the column vector of all ones. $\mathbf{\Pi}_0$ and $\mathbf{\Pi}_1$ are diagonal matrices with $\boldsymbol{\pi}_0 = [\sqrt{E_1}\bar{h}_1, \sqrt{E_2}\bar{h}_2, \dots, \sqrt{E_M}\bar{h}_M]^T$ and $\boldsymbol{\pi}_1 = [\sqrt{E_{PU}E_1}\bar{h}_1 h_1, \sqrt{E_{PU}E_2}\bar{h}_2 h_2, \dots, \sqrt{E_{PU}E_M}\bar{h}_M h_M]^T$ on their diagonals, respectively. The aggregate spectrum sensing observations at the FC are hence $\mathbf{Y} = [\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(K)]^T$.

2.2. Energy measuring and soft fusing

Soft fusion of the collected sensing observations is carried out at the FC. The FC first measures the received signal

energies from the M relays,

$$\mathbf{Z} = \text{vecdiag}(\mathbf{Y}^T \mathbf{Y}) = \begin{cases} \mathbf{Z}_0, & \mathcal{H}_0, \\ \mathbf{Z}_1, & \mathcal{H}_1, \end{cases} \quad (5)$$

where the function $\text{vecdiag}(\mathbf{X})$ creates a column vector whose elements are the main diagonal elements of matrix \mathbf{X} , test statistics $\mathbf{Z}_0 = [Z_{0,1}, Z_{0,2}, \dots, Z_{0,M}]^T$ and $\mathbf{Z}_1 = [Z_{1,1}, Z_{1,2}, \dots, Z_{1,M}]^T$ are the received signal energies captured within the sensing interval T_S and the frequency bandwidth W .

The captured PU signal energies in test statistics \mathbf{Z}_1 can be represented by the sum of K samples [14]

$$\begin{aligned} \theta_i &= \frac{1}{2W} \sum_{k=1}^K |y_i(k) - \tilde{n}_{\text{FC},i}(k)|^2 \\ &= E_{PU} E_i |h_i|^2 |\bar{h}_i|^2 T_S \\ &= \gamma_i N_{0,i} K, \end{aligned} \quad (6)$$

where $y_i(k)$ and $\tilde{n}_{\text{FC},i}(k)$ are the received signal and noise samples at time k under hypothesis \mathcal{H}_1 , respectively. $N_{0,i}$ is the equivalent one-sided noise power spectral density corresponding to the i -th relayed signal, γ_i is the PU signal-to-noise ratio (SNR) of the i -th relayed signal. According to (3), we can summarize the noise power densities as

$$N_{0,i} = (E_i |\bar{h}_i|^2 \sigma_i^2 + \sigma_{\text{FC}}^2) / W, \quad i \in \{1, 2, \dots, M\}. \quad (7)$$

When K is asymptotically large (e.g., larger than 100) [15], we can well approximate the test statistics \mathbf{Z} as normal distributed variables, according to the central limit theorem (CLT), with means and variances [6]

$$\begin{cases} \mu_{0,i} = E[Z_{0,i}] = N_{0,i} K, \\ \delta_{0,i}^2 = \text{Var}[Z_{0,i}] = N_{0,i}^2 K. \end{cases} \quad \mathcal{H}_0, \quad (8)$$

$$\begin{cases} \mu_{1,i} = E[Z_{1,i}] = N_{0,i} K + \theta_i, \\ \delta_{1,i}^2 = \text{Var}[Z_{1,i}] = N_{0,i}^2 K + 2N_{0,i} \theta_i. \end{cases} \quad \mathcal{H}_1. \quad (9)$$

Based on \mathbf{Z} , by allocating different weight coefficients to them and combining them all, the FC fuses the M observations of \mathbf{Z} into a global test statistic,

$$Z_c = \sum_{i=1}^M \omega_i Z_i = \boldsymbol{\omega}^T \mathbf{Z}, \quad (10)$$

where $\boldsymbol{\omega} = [\omega_1, \omega_2, \dots, \omega_M]^T$ is the weighting coefficients satisfying $\|\boldsymbol{\omega}\|_2^2 = 1$, $\omega_i \geq 0$. The combining weight for the signal from a particular SU represents its contribution to the global decision. Consequently, the global test statistics Z_c has means and variances given by

$$\bar{Z}_c = E[Z_c] = \begin{cases} \boldsymbol{\omega}^T \mathbf{u}_0, & \mathcal{H}_0, \\ \boldsymbol{\omega}^T \mathbf{u}_1, & \mathcal{H}_1, \end{cases} \quad (11)$$

$$\text{Var}[Z_c] = \begin{cases} \sum_{i=1}^M \delta_{0,i}^2 \omega_i^2 = \boldsymbol{\omega}^T \boldsymbol{\Sigma}_0 \boldsymbol{\omega}, & \mathcal{H}_0, \\ \sum_{i=1}^M \delta_{1,i}^2 \omega_i^2 = \boldsymbol{\omega}^T \boldsymbol{\Sigma}_1 \boldsymbol{\omega}, & \mathcal{H}_1, \end{cases} \quad (12)$$

where vectors of means $\mathbf{u}_0 = [u_{0,1}, u_{0,2}, \dots, u_{0,M}]^T$ and $\mathbf{u}_1 = [u_{1,1}, u_{1,2}, \dots, u_{1,M}]^T$; $\boldsymbol{\Sigma}_0$ and $\boldsymbol{\Sigma}_1$ are diagonal matrices with $\boldsymbol{\delta}_0^2 = [\delta_{0,1}^2, \delta_{0,2}^2, \dots, \delta_{0,M}^2]^T$ and $\boldsymbol{\delta}_1^2 = [\delta_{1,1}^2, \delta_{1,2}^2, \dots, \delta_{1,M}^2]^T$ on the diagonals, respectively. It is worth noting that the statistics \mathbf{Z} do not have to be conditionally independent though hereby we utilize the independent case for the illustration purpose, i.e., with $\boldsymbol{\Sigma}_0$ and $\boldsymbol{\Sigma}_1$ diagonal. If the elements of \mathbf{Z} are correlated with each other, then the covariance matrices $\boldsymbol{\Sigma}_0$ and $\boldsymbol{\Sigma}_1$ are generally non-diagonal but the subsequent analysis will continue to hold.

Given a global threshold λ at the FC, the probabilities of false alarm and detection in cooperative spectrum sensing are, respectively,

$$P_{\text{FA}} = Q\left(\frac{\lambda - \mathbf{u}_0^T \boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^T \boldsymbol{\Sigma}_0 \boldsymbol{\omega}}}\right), \quad P_{\text{D}} = Q\left(\frac{\lambda - \mathbf{u}_1^T \boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^T \boldsymbol{\Sigma}_1 \boldsymbol{\omega}}}\right), \quad (13)$$

where $Q(x) = \int_x^{+\infty} \exp(-t^2/2) dt / \sqrt{2\pi}$.

3. Deflection coefficient maximization based optimal fusion

3.1. Normal deflection coefficient maximization

For a cooperative spectrum sensing algorithm, the main metric of sensing performance is either the maximization of the detection probability for a target false alarm probability or minimization of the false alarm probability for a target detection probability. Setting the threshold λ for a desired probability of false alarm $P_{\text{FA,DES}}$, we obtain the probability of detection with the Neyman–Pearson criterion,

$$P_{\text{D}} = Q\left(\frac{Q^{-1}(P_{\text{FA,DES}}) \sqrt{\boldsymbol{\omega}^T \boldsymbol{\Sigma}_0 \boldsymbol{\omega}} + \mathbf{u}_0^T \boldsymbol{\omega} - \mathbf{u}_1^T \boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^T \boldsymbol{\Sigma}_1 \boldsymbol{\omega}}}\right), \quad (14)$$

where $Q^{-1}(\cdot)$ is the inverse function of $Q(\cdot)$.

From (11) and (12) it is clear that the weight vector $\boldsymbol{\omega}$ plays an important role in determining the probability density functions (PDFs) of the global test statistic Z_c under both hypotheses. To measure the effects of the PDFs on the detection performance, we introduce a normal deflection coefficient (NDC) [16]

$$\begin{aligned} d_{\text{NDC}}^2(\boldsymbol{\omega}) &= \frac{(E[Z_c | \mathcal{H}_1] - E[Z_c | \mathcal{H}_0])^2}{\text{Var}(Z_c | \mathcal{H}_0)} \\ &= \frac{(\boldsymbol{\Theta}^T \boldsymbol{\omega})^2}{\boldsymbol{\omega}^T \boldsymbol{\Sigma}_0 \boldsymbol{\omega}}, \end{aligned} \quad (15)$$

where $\boldsymbol{\Theta} = [\theta_1, \theta_2, \dots, \theta_M]^T$. The deflection coefficient $d_{\text{NDC}}^2(\boldsymbol{\omega})$ provides a good measure of the detection performance, because the covariance matrix $\boldsymbol{\Sigma}_0$ under hypothesis \mathcal{H}_0 is used to characterize the variance-normalized distance between the centers of the two conditional PDFs of Z_c under \mathcal{H}_0 and \mathcal{H}_1 . In the subsequent subsection, we will find

the optimal soft fusion solutions based on the modified DC (MDC), which employs Σ_1 under hypothesis \mathcal{H}_1 to fulfill the task of variance-normalization in (15).

The optimal weight vector $\omega_{opt,NDC}$ is defined as the one that maximizes the distance $d_{NDC}^2(\omega)$,

$$\omega_{opt,NDC} = \arg \max_{\omega} d_{NDC}^2(\omega). \quad (16)$$

By solving the equation $\partial d_{NDC}^2(\omega)/\partial \omega = 0$, we obtain

$$\begin{aligned} \omega_{opt,NDC}^* &= \frac{\omega^T \Sigma_0 \omega}{\omega^T \Theta} \Sigma_0^{-1} \Theta \\ &= \beta_{NDC} \Sigma_0^{-1} \Theta, \end{aligned} \quad (17)$$

where β_{NDC} is a scaling factor determined by ω , but it does not affect the detection performance in (14). By setting β_{NDC} to 1 and normalizing each weighting coefficient, we obtain the optimal weighting vector

$$\omega_{opt,NDC} = \omega_{opt,NDC}^* / \|\omega_{opt,NDC}^*\|_2. \quad (18)$$

According to the Schwarz inequality, we obtain another method in derivation of the optimal weight vector given in (17),

$$\begin{aligned} [\omega^T (\mathbf{u}_1 - \mathbf{u}_0)]^2 &= [\omega^T \Sigma_0^{1/2} \Sigma_0^{-1/2} (\mathbf{u}_1 - \mathbf{u}_0)]^2 \\ &= (\omega^T \Sigma_0 \omega) [(\mathbf{u}_1 - \mathbf{u}_0)^T \Sigma_0^{-1} (\mathbf{u}_1 - \mathbf{u}_0)] \\ &= (\omega^T \Sigma_0 \omega) \Theta^T \Sigma_0^{-1} \Theta, \end{aligned} \quad (19)$$

where the maximum of NDC is achieved as $\Theta^T \Sigma_0^{-1} \Theta$, and the equation is satisfied only when the optimal weights $\omega_{opt,NDC}^*$ for combining the M observations is

$$\begin{aligned} \omega_{opt,NDC}^* &= \eta_{NDC} \Sigma_0^{-1} (\mathbf{u}_1 - \mathbf{u}_0) \\ &= \eta_{NDC} \Sigma_0^{-1} \Theta, \end{aligned} \quad (20)$$

where η_{NDC} is a constant imposing no effect on $d_{NDC}^2(\omega)$ and thus can be set to 1. So far, the validity of the derived optimal weights $\omega_{opt,NDC}^*$ is proved in (17) and (19), respectively.

The detection performance of the NDCM based cooperative sensing in the Neyman–Pearson framework is then given by

$$P_D = Q \left(\frac{Q^{-1}(P_{FA,DES}) \sqrt{\Theta^T \Sigma_0^{-1} \Theta} - \Theta^T \Sigma_0^{-1} \Theta}{\sqrt{\Theta^T \Sigma_0^{-2} \Sigma_1 \Theta}} \right). \quad (21)$$

For a given $P_{FA,DES}$, P_D is maximized in the sense that the distance between the two PDFs of Z_c under hypotheses \mathcal{H}_0 and \mathcal{H}_1 is enlarged to the maximum by $\omega_{opt,NDC}$. In other words, the derived weights $\omega_{opt,NDC}$ is optimal in terms of NDCM. However, in the Neyman–Pearson framework, $\omega_{opt,NDC}$ is only a sub-optimal solution, since the deflection coefficient $d_{NDC}^2(\omega)$ is only part of the global optimization function

$$J(\omega) = \frac{Q^{-1}(P_{FA,DES}) \sqrt{\omega^T \Sigma_0 \omega} + \mathbf{u}_0^T \omega - \mathbf{u}_1^T \omega}{\sqrt{\omega^T \Sigma_1 \omega}}, \quad (22)$$

where $J(\omega)$ is the optimization function obtained from (14). Therefore, the maximization of the NDC might not reach the upper bound of the detection probability, given a required constant false alarm probability (CFAP). Nevertheless, to the best of the authors' knowledge, there is currently no close-form solution of maximizing the function $J(\omega)$ in the literature.

The detection performance P_D in (21) is actually a probability conditioned on the PU signal energy vector Θ , which is a composite random variable vector determined by channel gains h_i and \bar{h}_i . Therefore, the statistically averaged P_D is

$$\bar{P}_D = \int_{\mathbf{G} \subset \mathbb{R}_+^M} P_D(\mathbf{X}) p_{\Theta}(\mathbf{X}) d\mathbf{X}, \quad (23)$$

where $p_{\Theta}(\cdot)$ is the joint PDF of the multi-variable vector Θ , \mathbf{G} is the subset of \mathbb{R}_+^M that contains all Θ vectors leading to the \mathcal{H}_1 decision, and \mathbb{R}_+^M is the positive M -dimensional vector space.

3.2. Modified deflection coefficient maximization

In this subsection, we investigate the MDCM based optimal weights for the soft fusion at the FC. Note that if Σ_0 in (15) is substituted by Σ_1 , we can easily obtain the MDC, which is first defined in [9] as

$$\begin{aligned} d_{MDC}^2(\omega) &= \frac{(E[Z_c | \mathcal{H}_1] - E[Z_c | \mathcal{H}_0])^2}{\text{Var}(Z_c | \mathcal{H}_1)} \\ &= \frac{(\Theta^T \omega)^2}{\omega^T \Sigma_1 \omega}. \end{aligned} \quad (24)$$

The derivations of optimal weights in the previous subsection still hold except that in terms of MDCM, we have

$$\omega_{opt,MDC} = \Sigma_1^{-1} \Theta. \quad (25)$$

The MDCM in (24) is actually a general Rayleigh quotient problem, where

$$\begin{aligned} d_{MDC}^2(\omega) &= R(\mathbf{R}_{\Theta}, \Sigma_1; \omega) \\ &\triangleq \frac{\omega^T \mathbf{R}_{\Theta} \omega}{\omega^T \Sigma_1 \omega}. \end{aligned} \quad (26)$$

The optimal weight vector $\omega_{opt,MDC}$ is defined as the one that maximizes $R(\mathbf{R}_{\Theta}, \Sigma_1; \omega)$

$$\omega_{opt,MDC} = \arg \max_{\omega} R(\mathbf{R}_{\Theta}, \Sigma_1; \omega). \quad (27)$$

The generalized Rayleigh quotient $R(\mathbf{R}_{\Theta}, \Sigma_1; \omega)$ in (26) can be reduced to the Rayleigh quotient $R(\mathbf{D}, \mathbf{C}^T \omega)$ through the transformation $\mathbf{D} = \mathbf{C}^{-1} \mathbf{R}_{\Theta} \mathbf{C}^{-T}$ [17], where \mathbf{C} is the Cholesky decomposition of matrix Σ_1 . It is well known that the Cholesky decomposition of Σ_1 is a decomposition of the

symmetric, positive-definite matrix Σ_1 into a lower triangular matrix and the transpose of the lower triangular matrix,

$$\Sigma_1 = \mathbf{C}\mathbf{C}^T. \tag{28}$$

Now we obtain the Rayleigh quotient as

$$\begin{aligned} R(\mathbf{R}_\Theta, \Sigma_1; \omega) &= R(\mathbf{D}, \mathbf{C}^T \omega) \\ &= \frac{\boldsymbol{\varpi}^T \mathbf{D} \boldsymbol{\varpi}}{\boldsymbol{\varpi}^T \boldsymbol{\varpi}}, \end{aligned} \tag{29}$$

where $\boldsymbol{\varpi} = \mathbf{C}^T \omega$. Notice that $R(\mathbf{A}, \mathbf{x}') = R(\mathbf{A}, \mathbf{x})$, for $\mathbf{x}' = c\mathbf{x}$, $\forall c \neq 0$, therefore we will solve for $\boldsymbol{\varpi}$ with a unit norm $\|\boldsymbol{\varpi}\|^2 = 1$,

$$\begin{aligned} \max \quad & \boldsymbol{\varpi}^T \mathbf{D} \boldsymbol{\varpi} \\ \text{s.t.} \quad & \boldsymbol{\varpi}^T \boldsymbol{\varpi} = 1. \end{aligned} \tag{30}$$

By using the scalar Lagrange multiplier $\lambda_L \in \mathbb{R}$, we obtain

$$L(\boldsymbol{\varpi}) = \boldsymbol{\varpi}^T \mathbf{D} \boldsymbol{\varpi} + \lambda_L (\boldsymbol{\varpi}^T \boldsymbol{\varpi} - 1), \tag{31}$$

where by solving $\partial L(\boldsymbol{\varpi})/\partial \boldsymbol{\varpi} = 0$, we can easily find the optimal weight vector $\boldsymbol{\varpi}_{opt,MDC}$ as the eigenvector corresponding to the largest eigenvalue of \mathbf{D} , i.e., $eig_{max}(\mathbf{D}) = \mathbf{C}^{-1} \Theta$, where function $eig_{max}(\mathbf{D})$ returns the unit eigenvector of the maximum eigenvalue belonging to \mathbf{D} . Now, we can obtain the optimal weights $\boldsymbol{\omega}_{opt,MDC} = \Sigma_1^{-1} \Theta$, which is identical to the result in (25) and the solution in (61) of [9].

The normalized optimal weights $\boldsymbol{\omega}_{opt,MDC}$ maximizes the corresponding probability of detection in terms of MDCM, which is given as

$$P'_D = Q \left(\frac{Q^{-1}(P_{FA,DES}) \sqrt{\Theta^T \Sigma_1^{-2} \Sigma_0^{-1} \Theta} - \Theta^T \Sigma_1^{-1} \Theta}{\sqrt{\Theta^T \Sigma_1^{-1} \Theta}} \right), \tag{32}$$

where P'_D is still conditioned on the PU signal energies Θ and the statistically averaged probability of detection \bar{P}'_D can be easily obtained, similar to (23).

4. Implementation

4.1. Optimal global threshold setting method

In previous sections, the global threshold at the FC is determined in the Neyman–Pearson framework, based on the given desired probability of false alarm. In this subsection, we derive the optimal global threshold in accordance with Bayesian cost-effectiveness.

The philosophy behind setting the optimal threshold is that the *a priori* knowledge of the received signal under both hypotheses \mathcal{H}_0 and \mathcal{H}_1 can be exploited in setting the threshold dynamically, whereas in the Neyman–Pearson framework the threshold is set to be fixed, only depending on the statistical properties of the noise under hypothesis \mathcal{H}_0 .

Since the Neyman–Pearson threshold does not take into account the PU signal strength, the probability of false alarm is subject to a lower bound determined by the desired probability of false alarm, which can not be overcome no matter how large the SNR is. As for the optimal threshold, the probability of false alarm can be significantly improved, because the threshold is dynamically adjusted according to the current PDFs of the received signal under both hypotheses. In other words, if the distance between the centers of the two PDFs under \mathcal{H}_0 and \mathcal{H}_1 is large, an intuitive way is to set the threshold relatively large to suppress the false alarm and in the meantime result in no miss detection of the PU signal.

For ED based cooperative spectrum sensing, we define the optimal global threshold at the FC is the one capable of minimizing the simplified Bayesian risk

$$\begin{aligned} \mathfrak{R} &= E[C] \\ &= C_{FA} P(\mathcal{H}_0) P_{FA} + C_{MISS} P(\mathcal{H}_1) P_{MISS}, \end{aligned} \tag{33}$$

where C is the system cost, $P_{MISS} = 1 - P_D$ is the probability of miss detection, and C_{FA} and C_{MISS} are the system costs of false alarm and miss detection, respectively. It is worth noting that as a simplified Bayesian risk, the costs of correctly identifying PU signal and the spectrum opportunity are both set to 0.

To further simplify the optimality criterion and without loss of generality, we set both of $C_{FA} P(\mathcal{H}_0)$ and $C_{MISS} P(\mathcal{H}_1)$ to 1 and consequently the Bayesian risk \mathfrak{R} shrinks to a detection error function $F = P_{FA} + P_{MISS}$ [18]. The optimal threshold λ_{opt} for minimizing F is

$$\begin{aligned} \lambda_{opt} &= \arg \min_{\lambda} F(\lambda) \\ &= \arg \min_{\lambda} (P_{FA}(\lambda) + P_{MISS}(\lambda)). \end{aligned} \tag{34}$$

By solving the equation $\partial F/\partial \lambda = 0$ and referring to (13), we obtain

$$\frac{1}{\Psi_0} \exp \left(-\frac{(\lambda_{opt} - \Phi_0)^2}{2\Psi_0^2} \right) = \frac{1}{\Psi_1} \exp \left(-\frac{(\lambda_{opt} - \Phi_1)^2}{2\Psi_1^2} \right), \tag{35}$$

where $\Psi_0 = \sqrt{\omega^T \Sigma_0 \omega}$, $\Psi_1 = \sqrt{\omega^T \Sigma_1 \omega}$, $\Phi_0 = \omega^T \mathbf{u}_0$, $\Phi_1 = \omega^T \mathbf{u}_1$.

Eq. (35) means that in fact we choose the intersection of the two Gaussian distributions of the global test statistic Z_c under hypotheses \mathcal{H}_0 and \mathcal{H}_1 as the optimal threshold. Taking the natural logarithm of both sides of (35) and rearranging the terms, we can obtain the optimal threshold by solving

$$\begin{aligned} & (\Psi_1^2 - \Psi_0^2) \lambda_{opt}^2 - 2(\Phi_0 \Psi_1^2 - \Phi_1 \Psi_0^2) \lambda_{opt} \\ & + \Phi_0^2 \Psi_1^2 - \Phi_1^2 \Psi_0^2 - 2\Psi_0^2 \Psi_1^2 \ln \left(\frac{\Psi_1}{\Psi_0} \right) = 0. \end{aligned} \tag{36}$$

Since (36) is a second order polynomial equation that can be easily solved, we obtain

$$\lambda_{opt} = \frac{\Phi_1 \Psi_0^2 - \Phi_0 \Psi_1^2 + \Psi_0 \Psi_1 \sqrt{\Gamma}}{\Psi_0^2 - \Psi_1^2}, \quad (37)$$

where $\Gamma = (\Phi_0 - \Phi_1)^2 + 2(\Psi_1^2 - \Psi_0^2) \ln(\Psi_1/\Psi_0)$.

So far, the optimal threshold in soft fusion schemes for minimizing the error detection function F is obtained. Note that according to (13), even as the optimal threshold λ_{opt} is a function of the optimal weight $\omega_{opt,NDC}$, the scaling factor β_{NDC} or η_{NDC} imposes no impact on the performance of P_{FA} and P_D , and therefore we can safely set them to 1.

4.2. Recursive estimate algorithm for weight-setting

The optimal weighting vector in (18) is mainly determined by the signal energy quantities $\{E_{PU}|h_i|^2\}_{i=1}^M$, under the assumption that the channel gains \bar{h}_i , the relay power E_i , the noise variances σ_i^2 and σ_{FC}^2 are readily available for the FC before the sensing operation begins. These assumptions are justified by the fact that each SU can perform noise power estimation between the consecutive sensing intervals, and the channel gains between the SUs and the FC can also be obtained accurately due to some pilot-aided channel estimations performed at the FC. Additionally, we assume the channel coherence time of \bar{h}_i is much larger than the channel estimation period, such that the FC could adaptively estimate the channel gains from the SUs with small overhead.

With the above assumptions, the parameters we need to identify for setting the weight vector are only the signal energy quantities $\{E_{PU}|h_i|^2\}_{i=1}^M$ for the NDCM method. As for MDCM, an additional estimation of Σ_1 is also required. To obtain PU signal energies hidden in the raw sensing data of the cooperative SUs, a simple yet effective method is employed hereafter. By introducing records of the PU's behaviors, the current sensing data \mathbf{Z} is categorized and stored in either a *Presence* or an *Absence* matrix for future reference, according to the current global decision. In other words, if it is decided that the current data \mathbf{Z} contains the PU signal energy, it will be stored in an M -by- L Presence matrix $\mathbf{Z}^{(P)}$ in a first-in-first-out (FIFO) manner; otherwise it is stored in an M -by- L Absence matrix $\mathbf{Z}^{(A)}$ and meanwhile a zero column vector is pushed into $\mathbf{Z}^{(P)}$ which means no PU signal is present at the current time. The estimates of $E_{PU}|h_i|^2$ for the current statistic $Z_{c,j}$ are calculated via simple arithmetic averaging operations

$$\begin{aligned} \tilde{E}_{PU}|\tilde{h}_{i,j}|^2 &= \frac{1}{E_i|\tilde{h}_{i,j-1}|T_S} \frac{1}{L} \sum_{m=j-L}^{j-1} |Z_{i,m}^{(P)} - Z_{i,m}^{(A)}| \\ &= \frac{L-1}{L} \Delta_{i,j-1}^{(P)} + \frac{|Z_{i,j-1}^{(P)} - Z_{i,j-1}^{(A)}|}{LE_i|\tilde{h}_{i,j-1}|T_S}, \end{aligned} \quad (38)$$

where j is the time index of the current sensing data \mathbf{Z} , L is the reference matrix depth, and $\Delta_{i,j}^{(P)}$ is the estimate of

$E_{PU}|h_i|^2$ at instant j . An implicit assumption behind (38) is that the channels between the PU and the M SUs are slowly varying, which means the length L should be set sufficiently shorter than the channel varying interval.

Similarly, if the variances of the test statistics \mathbf{Z} under \mathcal{H}_0 need to be estimated at time j , we have

$$\begin{aligned} \tilde{\delta}_{0,i,j}^2 &= \frac{1}{L} \sum_{m=j-L}^{j-1} |(Z_{i,m}^{(A)})^2 - \tilde{u}_{0,i,j}^2| \\ &= \frac{L-1}{L} \Delta_{i,j-1}^{(A)} + \frac{1}{L} |(Z_{i,j-1}^{(A)})^2 - u_{0,i,j-1}^2|, \end{aligned} \quad (39)$$

where $\Delta_{i,j-1}^{(A)} = \tilde{\delta}_{0,i,j-1}^2$, $\tilde{u}_{0,i,j} = (1/L) \sum_{m=j-L}^{j-1} Z_{i,m}^{(A)}$.

Note that practically the matrix depth L in (39) can be set quite much larger than that in (38) to achieve more accurate estimation of the test statistics' variances under \mathcal{H}_0 .

With the estimated PU signal strength vector $\tilde{\Theta}$ and test statistic variance $\{\tilde{\delta}_{0,i,j}^2\}_{i=1}^M$, we can construct the estimated diagonal covariance matrix $\tilde{\Sigma}_1$, according to (9), to set the weights for the MDCM method. It is obvious that the estimate algorithm for weight setting in MDCM is more complex than that in NDCM.

4.3. Complexity analysis

Practically, the M cooperative SUs can work in either AAF or decode-and-forward (DAF) manner. Undoubtedly, the AAF scheme is much more complex than the DAF scheme and requires more control channel bandwidth, because the SUs operating in DAF will only relay the measured energies of the received signals to the FC. However, the AAF scheme has some potential benefits in improving the sensing performance by employing some signal processing techniques at the FC on the basis of the received signal samples, e.g., the BCED in [12]. In order to relay these received signal samples or measured signal energies, quantization operations need to be performed at each SU no matter either AAF or DAF scheme is adopted for relaying. Since investigating the impact of SNR loss in the quantization operation is beyond the scope of this paper, we still use the unquantized samples for signal representations in the previous derivations.

The system complexity of the proposed cooperative sensing scheme is comprised of two parts:

- (1) The control channel bandwidth W_{CTRL} required for relaying the sensing observations.
- (2) The required memory units of the recursive estimate algorithm for setting the optimal fusion weights.

Within each sensing interval T_S , there are K samples at each SU to be conveyed to the FC over the control channel. Suppose directional antennas are used at the SUs and the FC and each SU quantizes the received signal samples with N

Table 1. Complexity of the proposed optimal fusion scheme.

K	Absolute W_{CTRL} (kHz)	Relative W_{CTRL} (%)
100	200	2
200	400	4
400	800	8

bits/sample, the total number of sensing bits is NK for each SU. Because of the cooperation between the SUs and the FC, high order modulation schemes, e.g., QPSK and 16QAM, can be utilized on the control channel. Consequently, we have the required control channel bandwidth as

$$W_{CTRL} = \frac{1}{\xi} R_b = \frac{KN}{\xi T_R} = \frac{2T_S W N}{\xi T_R}, \quad (40)$$

where ξ is the spectral efficiency of the modulation scheme, R_b is the bit rate, and T_R is the time used by one SU to relay its sensing observations. Suppose $W = 10\text{MHz}$, $T_S = 10\mu\text{s}$, $T_R = 1\text{ms}$, $N = 8$, and $\xi = 4$ (for 16QAM with Nyquist minimum bandwidth), the required W_{CTRL} is given in Table 1.

The required W_{CTRL} in (40) is applied to the narrowband control channel case. An alternative to the narrowband control channel is to relay the sensing samples via low power UWB signaling [19,20], which possesses ample bandwidth for the AAF scheme. If the narrowband control channel has a very stringently limited bandwidth, the AAF can be easily reduced to the DAF scheme and the previous methods of deriving the optimal weights remain valid.

As for the proposed recursive estimate algorithm, it is fairly easy to implement. The number of required memory units for storing the sensing data is determined by the number of cooperative SUs M and the matrices depth L . Since the estimate algorithm is a simple arithmetical averaging operation over a finite observations, the computation load is very low and is linearly proportional to the product $2ML$.

5. Simulations and discussions

In this section, the proposed optimal cooperative spectrum sensing scheme is evaluated via simulations. The basic parameters are fixed and set as $T_S = 10\mu\text{s}$, $W = 10\text{MHz}$, $M = 10$, and $L = 16$. Each simulation consists of 10^5 iterations. The channel gains between the M SUs and the target PU are generated according to a complex normal distribution, which suggests that the PU signal is undergoing independent and identically distributed (i.i.d) Rayleigh fading before reaching the M SUs. In simulation, we suppose that the variances $\{\delta_{0,i}^2\}_{i=1}^M$ are distributed around an average level δ^2 , with a deviation d , which is normally distributed as $N(0, D\delta^2)$. D indicates the location difference factor in the CR network and is set to 20% in simulations.

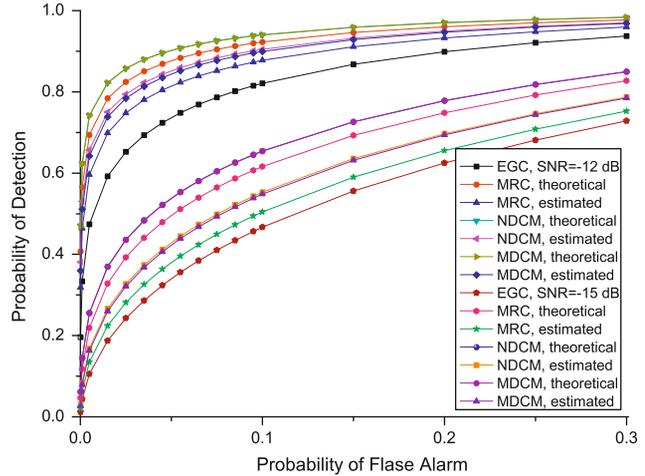


Fig. 1. ROC performance of the proposed optimal fusion scheme.

For simplicity, we assume that the PU signal power E_{PU} and the channel gains $\{h_i\}$ have constant values within each sensing interval T_S , provided that T_S was sufficiently small. This assumption is reasonable and can be encountered in a realistic scenario, where the variation of the dynamic radio environment is reflected in the channel gains' variation over a relatively large time-scale.

Fig. 1 demonstrates the receiver operating characteristics (ROC) of the proposed cooperative sensing scheme. It is found that the NDCM and MDCM based soft fusions have almost the same theoretical performance, whereas practically the performance of NDCM is slightly better than that of MDCM, because the MDCM solution introduces estimates of the PU signal strength and test statistic variance into the estimated covariance matrix simultaneously. As expected and shown, given the same number of SUs, the theoretical optimal weights $\omega_{opt,NDC}$ outperform the estimated ones, with non-trivial difference. This performance degradation is resulted by the absence of *a priori* knowledge of the PU signal and noise variance, because the task of extracting the PU signal energy from the noise in a low SNR environment is extremely challenging. However, this performance deterioration can be sufficiently compensated by increasing the number of cooperative SUs and obtaining an optimal L which is adaptively adjusted according to the channel variation speed. Compared to the MRC and EGC schemes, the proposed optimal fusion scheme improves the cooperative sensing performance significantly. Additionally, with the increase of SNR, the performance of the estimated weights effectively approaches that of the theoretical weights.

Fig. 2 gives the detection error probability in accordance with the proposed optimal global threshold setting method. We only use the NDCM based soft fusion for performance comparisons of the Neyman–Pearson threshold and the optimal threshold. As shown, the proposed optimal threshold is capable of reducing the detection error probability when the average SNR is increased, whereas the Neyman–Pearson

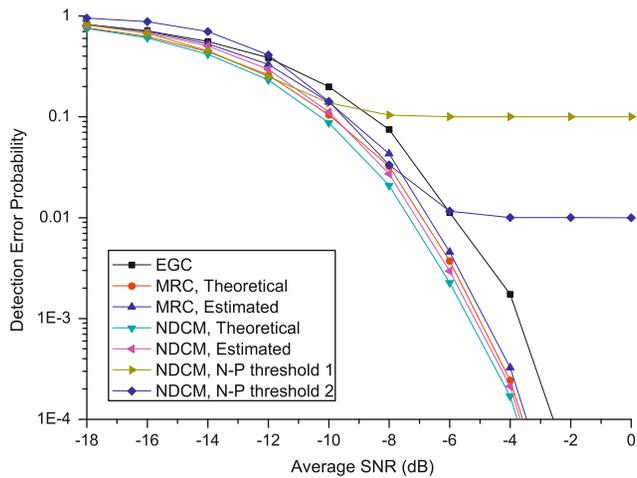


Fig. 2. Detection error probabilities of the proposed optimal fusion scheme.

thresholds (shown as thresholds 1 and 2 in Fig. 2) can only approach the desired $P_{FA,DES}$ of 0.1 and 0.01 no matter how large the SNR is increased. The superiority of the proposed optimal threshold is achieved by utilizing information of the PU signal in setting the threshold dynamically. With the help of the optimal threshold, the chance of capturing the spectrum opportunities is increased and the chance of detecting the PU signal is also maximized. Moreover, the proposed NDCM based weighting scheme is also verified to outperform the MRC and EGC schemes for both theoretical and estimated weights when the optimal threshold is used.

6. Conclusions

In this paper, deflection coefficient maximization criterion based optimal soft fusion scheme is proposed for cooperative spectrum sensing. In terms of DCM, we give close-form mathematical solutions to the NDCM and MDCM problems for improving the overall detection probability in the CR network. To implement the developed scheme, an optimal global threshold setting strategy and a recursive weight setting scheme are also proposed. As illustrated by our analysis and simulations, the proposed optimal soft fusion scheme outperforms the conventional MRC and EGC schemes and yields significant improvements in spectrum sensing.

Acknowledgment

This research was supported by the MKE (Ministry of Knowledge Economy), Korea, under the ITRC (Information Technology Research Center) support program supervised by the IITA (Institute of Information Technology Assessment) (IITA-2009-C1090-0902-0019).

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