Application of fuzzy calculations for improving portfolio matrices

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A B S T R A C T

In this paper, we draw upon portfolio analysis to develop and implement a strategic decision making model. We consider the limitations and problems of classic portfolio analysis approach and resolve these problems by using fuzzy set theory. In the proposed method, both internal and external factors are evaluated in linguistic terms and in terms of fuzzy triangular numbers. The fuzzy numbers are fed into an industry attractiveness-business analysis matrix. The novelty of the proposed approach comes from the fact that fuzzy numbers are processed without using conventional methods, allowing for strategies to be ranked.

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1. Introduction

Strategic management is defined as the science of formulating, executing, and assessing those decisions that enable organizations to achieve their long-term goals [4]. The most important stage in this process deals with strategy formulation. In this stage, the organization’s mission is determined, and threats and opportunities are identified. Strengths and weaknesses are identified, long-term goals are determined, various strategies are proposed, and specific strategies are selected.

One of the most important instruments used in this stage is the business portfolio analysis matrix [3,4,8,13]. The matrix is composed of zones, which represent predefined sets of strategies. The evaluation of the firm’s internal and external conditions results in a position (usually a point) in one of the strategy zones. The industry attractiveness-business strength matrix is the variation of the business portfolio analysis matrix that we have selected for the purposes of this research (see Fig. 1) [4].

The strategies represented by each zone are as follows:

(1) Cells 1, 2 and 4 (zone A) comprise strategies of growth and building, and also offensive strategies. These strategies are known as “intensive strategies,” i.e., market penetration, market development, product development, forward integration, backward integration and horizontal integration.

(2) Cells 3, 5 or 7 (zone B) call for the preservation of the status quo. In this zone, common strategies are market penetration and product development.

(3) Units located in cells 6, 8 or 9 (zone C) should divest.

In the above matrix, point calculations are usually based on inexact terms, but fuzzy-based methodologies have seldom been used. The application of fuzzy theory to the interpretation of portfolio matrices is recommended in [11,12]. In [11], the
industry attractiveness-business strength matrix is fuzzified. In [12] a fuzzy rule-based method is used to handle the growth-share matrix. The fuzzy concept also has been applied in optimal portfolio selection problems. For instance, in [9], the portfolio selection problem is investigated when the security returns contain both randomness and fuzziness. In [7], a fuzzy mathematical model is applied to a similar problem. In [1], a fuzzy random multi-objective quadratic programming method is examined and applied to the portfolio optimization problem. Finally, in [14], the analytic hierarchy process is applied to the portfolio selection problem.

Despite this extensive research, difficulties still persist: portfolio analysis recommends a strategy for each business unit based on its position in the company’s overall portfolio of businesses, according to known and accepted rules. This could lead to different strategy recommendations for business units that are very close to each other but on the opposite sides of boundaries in matrices. This last point can be construed as the first drawback of portfolio analysis. Also, the portfolio analysis suggests the same strategy choice for all business units in the same quadrant, regardless of their exact positions in the matrix. For example, one of the legitimate shortcomings of the growth-share matrix is that a “four-cell matrix based on high/low classification hides the fact that many businesses are in markets with an average growth rate and have relative market shares that are neither high nor low but in-between or intermediate. In which cells do these average businesses belong?” [12].

The expression of internal and external factors in the form of fuzzy numbers, instead of crisp numbers, provides a convenient answer to that question [5]. Therefore, one should consider an environment for the expression of strategies based on areas instead of points. Although some researchers have attempted to fuzzify the inputs of process of strategic planning [11,12], this paper considers both inputs and outputs of this process. Whereas the real nature of the environment for the implementation of strategies is ambiguous, we pursue an approach towards the expression of strategies in uncertain states. The ability to prioritize strategies is one of the outcomes of such an approach. Furthermore, by quantifying the factors as fuzzy triangular numbers, the evaluation of the factors is made more flexible, and both qualitative and quantitative aspects of the factors can be considered. The ambiguity and uncertainty of the factors can also be taken into account during the process of decision making. Therefore, our aim in this paper is to develop a flexible, adaptive, and realistic system to support the process of decision making.

2. Fuzzy sets application in industry attractiveness-business strength matrix

There are three main problems with the classical method of strategy extraction using the industry attractiveness-business strength matrix:

(1) The internal factor evaluation (IFE) matrix is used to evaluate the internal conditions of an organization. The IFE matrix comprises factors (strengths and weaknesses), weights (0.0–1.0), ratings (1–4: One for high weakness, two for low
weakness, three for low strength, and four for high strength), and the final weighted score is obtained by multiplying the ratings by the weights. The closer this number is to four, the stronger the internal factors, and vice versa.

A similar number can be obtained for the external factor evaluation (EFE) [4]. Assigning only four numbers (ratings) to each family of factors decreases the flexibility of the decision making model to express the organization situation. For instance, by assigning two numbers 3 and 4 to classify factors into high and low strengths, we limit our ability to consider the nuances of intermediate states. Similarly, in many other cases, assigning a number to a qualitative factor seems difficult and arbitrary [16,17].

(2) As noted earlier, the industry attractiveness-business strength matrix is composed of three adjacent strategic zones. Delineating these zones in the present form of the matrix is another problem. For example, assume that the final grade of one unit in internal factors is 2.95 and that of another unit is 3.05. Despite the fact that the two numbers are very close, these two units will be placed into two different zones [11].

(3) In the prevalent approaches to portfolio analysis, there is no method of prioritizing the proposed strategies. As shown in Fig. 2, the prevailing method determines just an area for each strategic business unit (SBU). Moreover, there is no criterion for the prioritization of the SBUs located in certain zones of the matrix. For example, there is no difference between SBUs 2, 3 and 4, which are in area B (squares 3, 5, 7).

3. The proposed method

3.1. Factors evaluation

Suppose that there are \( n \) SBUs to be evaluated considering \( m \) internal and \( k \) external factors. Suppose that \( W_i \) is the fuzzy weight number representing the importance of the internal factor \( F_i \) and \( D_{ij} \) is the desirability of the \( j \)th SBU regarding that factor, for all \( i \in I \{1, 2, \ldots, m\} \) and for all \( j \in J \{1, 2, \ldots, n\} \). The overall internal desirability of the SBU \( j \) is denoted by \( ID_j \).

The \( \alpha \)-cut of the overall internal desirability \( ID_j \) of a SBU \( j \) is calculated through fuzzy weight average (FWA) [15] for \( m \) desirability levels represented by the fuzzy numbers \( D_{1j}, D_{2j}, \ldots, D_{mj} \), with weights (fuzzy numbers) \( W_1, W_2, \ldots, W_m \), as follows:

\[
ID_j^\alpha = \left( ID_{1j}^\alpha, ID_{2j}^\alpha \right).
\]

Fig. 2. There is no difference between SBUs 2, 3 and 4, which are all in area B.
where
\[
ID_{j,a}^x = \min \left( \frac{\sum_{i \in I} D_{j,a}^{x} w_i^x}{\sum_{i \in I} w_i^x} \right)
\]
and
\[
ID_{j,b}^x = \max \left( \frac{\sum_{i \in I} D_{j,b}^{x} w_i^x}{\sum_{i \in I} w_i^x} \right)
\]
for all \( j \in J \). Here,
\[
w_i^x \in [W_{i,a}^x, W_{i,b}^x] \forall i \in I \text{ and } \forall x \in (0, 1].
\]

Clearly, \( ID_{j,a}^x \) and \( ID_{j,b}^x \) represent the lower and upper limits, respectively of the \( \alpha \)-cut \( ID_{j,a}^{x} \); \( D_{j,a}^{x} \), \( W_{j,a}^{x} \), and \( W_{j,b}^{x} \) likewise represent the lower and upper limits of the \( \alpha \)-cut \( D_{j,a}^{x} \); \( W_{j,a}^{x} \), and \( W_{j,b}^{x} \) represent the lower and upper limits, respectively, of the \( \alpha \)-cut \( W_{j,a}^{x} \). The “min” and “max” operators take the minimum and maximum values that can be calculated through the combination of the \( w_i^x \) in all the possible ways. The minimum and maximum values in Eqs. (2) and (3) are always obtained by taking (real) values of the weight \( w_i^x \) equal to the extreme values, i.e., \( W_{i,a}^x \) or \( W_{i,b}^x \) of the \( \alpha \)-cut \( W_{i,a}^x \) for all \( i \in I \) and all \( x \in (0,1] \). Then the condition in Eq. (4) can be relaxed to
\[
w_i^x \in \{W_{i,a}^x, W_{i,b}^x\} \forall i \in I \text{ and } \forall x \in (0, 1].
\]

The same approach can be applied for \( ED_j \) (the overall external desirability of SBU\(_i\)) so two fuzzy numbers are obtained for evaluation of business strength and industry attractiveness; like the method described in [2]. In this paper, for the sake of simplicity, all fuzzy numbers are represented by fuzzy triangular numbers, and \( ID_j \) and \( ED_j \) are approximated by the following fuzzy triangular numbers:
\[
ID_j = \left( ID_{j,a}^0, ID_{j,a}^1, ID_{j,b}^1 \right),
\]
\[
ED_j = \left( ED_{j,a}^0, ED_{j,a}^1, ED_{j,b}^1 \right),
\]
where \( ID_{j,a}^1 \left( ED_{j,a}^1 \right) \) is equal to \( ID_{j,a}^1 \left( ED_{j,a}^1 \right) \).

\[\text{Fig. 3. Aggregation of internal and external factors and resulted rectangle.}\]
3.2. Analysis of industry attractiveness-business strength matrix

To extract strategies based on internal and external factors, it is necessary to aggregate the membership functions of triangular numbers. For this purpose, we can define specific $\alpha$-cuts on fuzzy numbers [18]. According to Fig. 3, by projecting the intersection of $\alpha$-cuts onto the industry attractiveness-business strength matrix, a rectangle is obtained.

As shown in Fig. 4, using the concept of $\alpha$-cut, we can reduce the ambiguity in the possible positions of strategic business units in the matrix.

To identify the relative priority of strategic zones, we calculate the percentage of the rectangle in each zone. Obviously, the zone containing the maximum percentage of the rectangle determines the strategies to be adopted. For example, in Fig. 4, the prioritization of areas is as follows:

![Diagram showing projection of fuzzy numbers with and without $\alpha$-cut](image)

**Fig. 4.** Consideration of tendency by defining $\alpha$-cut.

| Table 1 |
|---|---|---|
| Internal factors | Factor weight | Factor evaluation |
| Strength  |
| Profit of company has increased in last year | Medium High | Very High |
| New products have succeeded | High | Medium |
| Market share has increased in Europe | Medium Low | High |
| Company has received a HACCP certificate | Low | Medium High |
| Weaknesses  |
| Sales of one existing product have decreased | Medium High | Medium Low |
| Cost of maintenance for old facilities and machines has increased | High | Low |
| Weakness in marketing | Very Low | Medium Low |
| Low present profit compared to standard level | Medium High | Very Low |
The percentage of the rectangle in each zone and the size of the rectangle depend on the value of $\alpha$. This value has a direct relation with the size and an inverse relationship with the differences between percentages placed in each area. Moreover, the robustness of the final judgment depends on these two factors (the area of the rectangle and the differences between percentages, which show the tendency of the fuzzy numbers toward each zone). Therefore, the value of $\alpha$ should be determined on the basis of a tradeoff between the two factors. The value of $\alpha$ is the solution of the following equation:

$$1 - \frac{NP_G}{P_G} = \frac{S_\alpha}{S_0},$$

where $P_G$ is the maximum percentage, $NP_G$ denotes the second highest percentage, $S_\alpha$ is the area of the rectangle when $\alpha = \alpha$, and $S_0$ is the area when $\alpha = 0$.

Table 2
External factor evaluation.

<table>
<thead>
<tr>
<th>External factors</th>
<th>Factor weight</th>
<th>Factor evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opportunity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of economical systems based on free market in south east Asia</td>
<td>Medium Low</td>
<td>High</td>
</tr>
<tr>
<td>Demand for products of the company in international markets has increased</td>
<td>High</td>
<td>Very High</td>
</tr>
<tr>
<td>High sensitivity for healthy food in international markets</td>
<td>Very High</td>
<td>Very High</td>
</tr>
<tr>
<td>Duty free laws in some countries</td>
<td>Very Low</td>
<td>Medium High</td>
</tr>
<tr>
<td>Threats</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compared to company income, production costs are increasing at a higher rate</td>
<td>Medium Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Low market share compared to other companies</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Uncertainty of consumption market in southeast Asia</td>
<td>Medium High</td>
<td>Very Low</td>
</tr>
<tr>
<td>Reluctance of some customers to consume ready-made foods</td>
<td>Very High</td>
<td>Low</td>
</tr>
</tbody>
</table>

Fig. 5. Overall desirability levels for the evaluation of SBUs.
4. A case illustration

In order to evaluate the applicability of the proposed algorithm, we implemented it in a strategic planning process for a food company in Iran [19]. The company is a successful player in the food industry in the Middle East.

4.1. Factors evaluation

The IFE and EFE matrices are given in Tables 1 and 2. Each factor is evaluated by two linguistic terms. There are different methods to convert linguistic terms to fuzzy numbers [18]. We convert the linguistic terms to fuzzy numbers by the scale shown in Fig. 5. As we can see, for each linguistic term, there is an equivalent fuzzy triangular number, (0, 0, 0.166) for very low, (0, 0.166, 0.334) for low and etc.

According to (2) and (3), the final result of the internal factor evaluation in the form of a triangular fuzzy number is as follows:

![Internal factors evaluation diagram](image)

Fig. 6. Aggregation of internal and external factors and resulting rectangle.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\text{Percentage of the rectangle in each zone}$</th>
<th>$S_x/S_0$</th>
<th>$1 - \frac{W_r}{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cell 2 Zone A</td>
<td>Cell 3 Zone B</td>
<td>Cell 5 Zone B</td>
</tr>
<tr>
<td>0.00</td>
<td>2.09</td>
<td>0.31</td>
<td>79.48</td>
</tr>
<tr>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
<td>85.78</td>
</tr>
<tr>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>90.80</td>
</tr>
<tr>
<td>0.30</td>
<td>0.00</td>
<td>0.00</td>
<td>97.25</td>
</tr>
<tr>
<td>&gt;0.3</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>
The final result of the external factor evaluation is

\[ \text{ID} = (0.279, 0.441, 0.557) \]

The final result of the external factor evaluation is

\[ \text{ED} = (0.436, 0.579, 0.673). \]

### 4.2. Industry attractiveness-business strength matrix analysis

By considering a specific value for \( \alpha \) (say \( \alpha = 0.3 \)), a rectangle is obtained by aggregating the internal and external fuzzy sets as shown in Fig. 6. The results for other values of \( \alpha \) are summarized in Table 3.

As can be seen in Table 3, \( \alpha = 0.1 \) is an approximate solution of Eq. (7), and about 85% of the resulting rectangle is located in zone B. Thus, with high confidence, strategies related to zone B are recommended, i.e., strategies for protecting the current situation in different ways, such as market penetration and product development. Zone C can be considered as the second priority.

Obviously, in this case, the usual methods of portfolio analysis do not make sense in light of the intensity of fitness in each zone (Fig. 7).

### 5. Conclusion

Portfolio analysis using a fuzzy approach has been the main focus of this paper. We have drawn on fuzzy methodology to extract strategies. Fuzzy methodology allows us to incorporate uncertainty into historical data and also to incorporate subjective/intuitive characteristics into the portfolio analysis models. Furthermore, the ambiguity and uncertainty of the factors can be taken into account during the process of decision making.

The industry attractiveness-business strength matrix is a variation of the business portfolio analysis matrix. In the usual non-fuzzy methods, the position of an organization is shown by a point on the matrix. In the proposed method, where fuzzy numbers were used in evaluations, we suggested selecting a region on the matrix. In this paper, we explained how to specify such a region, and accordingly how to extract strategies and determine their priority of them.

Although some researchers have attempted to fuzzify the inputs of the process of strategic planning, this paper attempts to consider both the inputs and outputs of this process. Whereas the real environment for implementation of strategies has
an ambiguous nature, the expression of strategies in an uncertain state is an important approach, which is pursued in this research. Therefore the major outcome of this approach is that it will allow the prioritization of strategies.

Last but not the least, our studies on solving a real problem in crisp [6] and fuzzy [10] senses show that differences between the extracted strategies is about fifty percent. Moreover, the fuzzy approach output is very sensitive to the value of α and the prioritization method. Thus investigating the validity and reliability of the fuzzy extracted strategies are the most important directions for future research.

References