

# Application of fuzzy sets with different t-norms in the interpretation of portfolio matrices in strategic management

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## Abstract

The possible application of fuzzy sets theory in strategic management to the problem of portfolio matrices analysis, used for strategy alternative(s) formulation and selection is described. The values of membership functions of input variables into portfolio matrices are combined with different t-norms: (a)  $T_M(x, y) = \min(x, y)$ ; (b)  $T_P(x, y) = xy$ ; (c) Sugeno's  $\lambda$  t-norms; (d) Hamacher's t-norm family, for investigating possible different results, but the recommended strategy selection remained the same for all applied t-norms. © 2000 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Strategic management can be defined as *the process by which an organization formulates objectives and is managed to achieve them* [3]. We refer to *the means used to achieve an organization's ends* as to strategy. Typically, the generic strategy's emphasis on one or two variables maximizes its visceral impact and intuitive appeal, but complicates its use. This is similar to the music: there are only eight notes in the musical scale, yet they can be arranged in an indefinite number of ways. The value of generic strategies lies more in the questions they provoke than in any simple effort to implement them independently.

Several methodologies have been developed to assist the selection of most adequate strategy, or rather the mix of strategies, in the domain of strategic management. One of them is known as business portfolio analyses. The term "interpretation of portfolios" is commonly used in the domain of strategic management to describe the two-phase activity: (1) analysis of the reasons and influences that resulted in exact organizations' position in matrices and (2) formulation of actions or steps that should be undertaken to improve the actual organizations' position and to reach the corporate goals. It could be viewed as a serious study of what have been done and what have still to be done for the organization to reach the desired success. Portfolio analysis recommends a strategy for each business unit based on its position in the company's overall portfolio of businesses according to known and accepted rules (see [1, 3, 12]). This could

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lead to different strategy recommendations for business units placed very closely to each other but on the opposite sides of delimiters in matrices, which is the first drawback of portfolio analyses. Also, the portfolio analyses suggest the same strategy choice for all business units placed in the same quadrant, regardless of the exact position in the matrix. One of the legitimate shortcomings for the growth-share matrix is expressed in [12] in the following way: “A four-cell matrix based on high–low classification hides the fact that many businesses are in markets with an average growth rate and have relative market shares that are neither high nor low but in-between or intermediate. In which cells do these average businesses belong?”

The domain of strategic management has already been recognized as a field convenient for the application of fuzzy sets theory, firstly, for the fuzziness of main concepts and terms, secondly, because the contexts of strategic management belong to the field of uncertainty and vagueness. Having this in mind, we incorporated fuzzy reasoning into portfolio analyses trying to make a qualitative shift in managers ability in strategic decision making. In the interpretation of portfolio matrices, we have leaned on the “sets as points” representation of fuzzy sets in  $n$ -dimensional unit cube, the representation proposed by Kosko [6]. We applied different t-norms, as proposed in [5] for the calculation of adequacy degrees attached to alternate strategy choices for investigating possible different results.

Sections 2 and 3 of this article introduce the relevant terms and concepts of fuzzy sets theory and give some basic definitions necessary for their understanding. The short review of “sets as points” representation of fuzzy sets is also the subject of Section 2. Section 4 explains in detail the main idea of fuzzy logic inclusion into portfolio analysis by Growth-share matrix. Each step of the fuzzy approach to strategy selection is described in one of Sections 4.1–4.4. Some other business portfolio matrices and their application for strategy selection are the subject of Section 5. The concluding remarks are given in Section 6.

## 2. Sets as points representation of fuzzy sets

The geometry of fuzzy sets involves both the domain  $X = \{x_1, x_2, \dots, x_n\}$  and the range  $[0, 1]$  of map-

pings  $\mu_A: X \rightarrow [0, 1]$ . Visualizing this geometry gives an answer to the question: “What does a fuzzy set look like?” The fuzzy set is a point in a cube. The set of all fuzzy sets equals the unit hyper cube  $I^n = [0, 1]^n$ . A fuzzy set as Kosko interpreted is any point in the cube  $I^n$ . So  $(X, I^n)$  defines the fundamental measurable space of finite fuzzy theory.

If we consider the fuzzy subsets of  $X$ , we can view the fuzzy subset  $A = (\frac{1}{3}, \frac{3}{4})$  as one of the continuum-many continuous-valued membership functions  $\mu_A: X \rightarrow [0, 1]$ . The first element  $x_1$  belongs to (fits in) subset  $A$  a little bit – to degree  $\frac{1}{3}$ . Element  $x_2$  has more membership than not at  $\frac{3}{4}$ . The *fit vector*  $(\frac{1}{3}, \frac{3}{4})$  represents  $A$ . The element  $\mu_A(x_i)$  equals the  $i$ th fit or *fuzzy unit value*. The “sets as points” view geometrically represents the fuzzy subset  $A$  as a point in  $I^2$ , the unit square.

## 3. Triangular norms

There is an abundance of literature that extensively explore the topic of triangular norms (see [5, 11]). We shall give only few definitions and notations in relation with this subject.

**Definition 1.** A triangular norm  $T$  (t-norm briefly) is a function  $T: [0, 1]^2 \rightarrow [0, 1]$  such that  
 (T1)  $T(x, y) = T(y, x)$  (commutativity),  
 (T2)  $T(x, T(y, z)) = T(T(x, y), z)$  (associativity),  
 (T3)  $T(x, y) \leq T(x, z)$  for  $y \leq z$  (monotonicity),  
 (T4)  $T(x, 1) = x$  (boundary condition).

The following are the most important t-norms:

$$T_M(x, y) = \min(x, y),$$

$$T_P(x, y) = xy,$$

$$T_L(x, y) = \max(0, x + y - 1),$$

$$T_W(x, y) = \begin{cases} \min(x, y) & \text{if } \max(x, y) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

**Definition 2.** A triangular conorm  $S$  (t-conorm) is a dual operation to corresponding t-norm  $T$ , such that

$$S(x, y) = 1 - T(1 - x, 1 - y).$$

There are several important parametrized families of t-norms and t-conorms. A majority of parametrized

families of t-norms is given by a corresponding family of additive generators.

Weber in 1983, see [5], suggested the use of the following t-norms  $(T_\lambda^{SW})_{\lambda > -1}$  and t-conorms  $(S_\lambda^{SW})_{\lambda > -1}$  to model the connectives AND and OR, respectively, for fuzzy sets. For  $\lambda > -1$ ,  $x, y \in [0, 1]$ , it is

$$T_\lambda^{SW}(x, y) = \max\left(0, \frac{x + y - 1 + \lambda xy}{1 + \lambda}\right),$$

$$S_\lambda^{SW}(x, y) = \min(x + y + \lambda xy, 1).$$

t-conorms  $S_\lambda^{SW}$ ,  $\lambda > -1$ , appeared already as “addition-rule” for Sugeno’s  $\lambda$ -fuzzy measures in Sugeno 1974, see [5], and therefore they are often called Sugeno’s additions, too.

Hamacher in 1978 investigated the axiomatic of continuous many-valued logical connectives AND and OR when the set of truth values was a unit interval. His main result can be reformulated in the following form: a continuous t-norm  $T$  is a rational function (that is  $T(x, y) = P(x, y)/Q(x, y)$  with polynomials  $P$  and  $Q$ ) if and only if  $T$  belongs to the family  $(T_k^H)_{k \in [0, \infty)}$ , where

$$T_k^H(x, y) = \frac{xy}{k + (1 - k)(x + y - xy)}$$

up to the case  $x = y = k = 0$ .

Put  $T_\infty^H = T_W$ . Then the family  $(T_k^H)_{k \in [0, \infty]}$  is continuous and decreasing in parameter  $k$ . Recall that  $T_1^H = T_P$ .

Often, as a limit member of parametrized families, the strongest t-norm  $T_M$  and the weakest t-norm  $T_W$  occur. A similar situation is in the case of t-conorms families. The reason for this fact is justified by the following theorem given by Dombi 1982, see [5].

**Theorem 1.** *Let  $f$  be an additive generator of a continuous Archimedean t-norm  $T$ , i.e.  $f : [0, 1] \rightarrow [0, \infty]$  is continuous, strictly decreasing mapping with  $f(1) = 0$ . Let  $\lambda \in (0, \infty)$  be a real constant. Then also  $f^\lambda$  is an additive generator. Let  $T_\lambda$  be a t-norm generated by  $f^\lambda$ . Then for each  $x, y \in [0, 1]$  it is*

$$\lim_{\lambda \rightarrow \infty} T_\lambda(x, y) = T_M(x, y)$$

and

$$\lim_{\lambda \rightarrow 0^+} T_\lambda(x, y) = T_W(x, y).$$

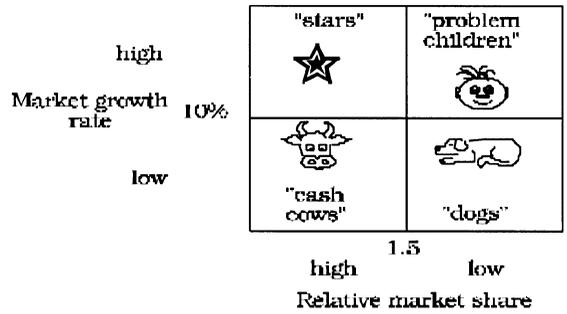


Fig. 1. Growth-share business portfolio matrix.

The fuzzy set intersection is defined by pairwise minimum of compared corresponding membership values, union by pairwise maximum and complementation by order reversal. Other t-norms and conorms can be used instead of min and max operators, respectively. The application of fuzzy sets theory in strategy evaluation and selection includes only the set intersection so in this paper we used the following t-norms:

- (a)  $T_M(x, y) = \min(x, y)$ ;
- (b)  $T_P(x, y) = xy$ ;
- (c) Sugeno’s  $\lambda$  t-norm:

$$T_\lambda^{SW}(x, y) = \max\left(0, \frac{x + y - 1 + \lambda xy}{1 + \lambda}\right), \quad \lambda > -1,$$

- (d) Hamacher’s t-norm:

$$T_k^H(x, y) = \frac{xy}{k + (1 - k)(x + y - xy)}, \quad k > 0.$$

#### 4. Application of fuzzy logic in strategic management portfolio analysis

One methodology developed to assist in the strategy evaluation and selection process is known as business portfolio analyses (see [1, 3, 12]). Among the various approaches of this type, the most popular seem to be the Growth-share matrix. In order to visually display an organization’s business portfolio, a four-quadrant grid is developed, as shown in Fig. 1. The horizontal axis in Fig. 1 indicates the market share of the business relative to its major competitor and characterizes the strength of the organization in that business. The vertical axis indicates the percent of growth in the market in the current year and characterizes

the attractiveness of the market for the business unit. According to their position in the matrix, business units identified in the corporation are called “stars”, “cash cows”, “problem children” or “dogs”. The four major strategic choices identified for business units by growth-share matrix are: (a) leadership strategy, (b) growth strategy, (c) divestiture strategy (d) divestiture and liquidation strategies. Herein these four strategy alternatives are briefly summarized.

*Leadership strategy:* The main strategic concern for a leader revolves around how to sustain a leadership position, perhaps becoming the dominant leader as opposed to a leader. Three contrasting strategic postures are open to industry leaders and dominant firms:

- (a) stay-on-the-offensive strategy, when the leaders try to be “first-movers” to build sustainable competitive advantage and a solid reputation as the leader;
- (b) fortify and defend strategy the goal of which is to hold onto present market share, strengthen current market position and protect whatever competitive advantage the firm has. For other specific defensive actions see [12]; and
- (c) follow-the-leader strategy with the goal of discouraging would-be challengers by quickly meeting all price cuts, countering with large-scale promotional campaigns when challengers make threatening moves to gain market share and offering better deals to the major customers of next-in-line or “maverick” firms.

*The growth strategy:* The growth strategy for runner-up firms building market share is based on:

- (a) becoming a lower-cost producer and using lower price to win customers from weak, higher-cost rivals and
- (b) using differentiation strategies based on quality, technological superiority, better customer service, best cost, or innovation.

*Divestiture strategies:* These strategies are for weak businesses. The most common one is the harvest strategy, that steers a middle course between preserving the status quo and exiting the business as soon as possible. Harvesting is a phasing down or endgame strategy: firm may gradually raise prices and cut promotional expenses, reduce quality in not so visible ways, decrease equipment maintenance, and the like (see [12]).

*Divestiture and liquidation strategies:* When a particular line of business loses its appeal, the most attractive solution usually is to sell it. Normally, such businesses should be divested as fast as is practical, unless time is needed to get them into better shape to sell. Of all the strategic alternatives, liquidation is the most unpleasant and painful, but still better than bankruptcy.

The application of different strategy alternatives depends on the business units position in portfolio quadrants, i.e. different strategies have to be pursued if both indicators, relative market share and market growth rate are high, than if both indicators are low or if one indicator is high and the other is low. Accordingly, “stars” should pursue leadership strategy and hold high market growth rate and high market share. “Cash cows” are those business units that make the greatest profit and produce extra cash that can be used for maintaining the “stars” position or improving the business in “problem children” business units in corporate overall portfolio. “Problem children” are those business units that still have the potential for increasing the relative market share, but there have to be some additional cash, free for necessary investments, usually taken from “cash cows”. Business units characterized as “dogs” are the less profitable ones and the best-strategy solution for them is to divest or liquidate, so the gained money could be directed to more manageable business units positioned in other quadrants in corporate portfolio matrix.

The lines that divide the growth-share matrix into four quadrants are somewhat arbitrarily set. A high growth rate is taken to be over 10%. The demarcation between high and low relative market share is set at 1,5. Such demarcations remind us the paradox of heap of sand: is it still a heap if we remove one grain of sand? How about two grains? Three? If we argue bivalently by induction, we eventually remove all grains and still conclude that a heap remains or that it has suddenly vanished. No single grain takes us from heap to non-heap. Physically, we experience degrees of occurrence. In terms of statements about the physical processes, we arrive at degrees of truth. Similarly, we cannot state that 10% is a high growth rate, while 9.9% is a low one.

The major difficulty with the use of the growth-share matrix is in determining market share in complex industries. Because of this, we have to replace the exact

numerical indicator by subjective approximation, i.e. fuzzy value.

In the article we describe the process of interpretation of portfolio matrix by the means of fuzzy sets theory through the following steps:

1. Representing, in a form of production rules, the dependencies between the values of input and output variables. The input variables are the fundamental parameters that determine the business unit's position in portfolio matrix. In the case of the growth-share matrix, these parameters are the market growth rate and the relative market share. The output variable is the recommended strategy choice.
2. Specifying the linguistic terms that represent the values of input variables (e.g. low, medium, high, etc.) and determining the appropriate membership functions for each of them.
3. Formulating the “rulebase” or bank of fuzzy associations sufficient for solving the strategy selection problem by means of portfolio analysis.
4. Calculating, through the parallel activation of all rules, the levels of adequacy for generic strategies to be pursued.

#### 4.1. Expressing the dependencies

Strategy selection using the portfolio matrix is based on two fundamental parameters that determine the business unit's position in the matrix. For each quadrant in the matrix there are predefined strategic choices ( $S_i$ ,  $i \in \{1, 2, 3, 4\}$ ) a business unit should pursue as the most adequate ones. For the growth-share matrix this domain knowledge can be expressed in a form of rules such as:

- (1) If the market growth rate is high and the relative market share is high, then the most adequate strategy choice is leadership strategy ( $S_1$ );
- (2) If the market growth rate is low and the relative market share is low then the most adequate strategy choices are divestiture and liquidation strategies ( $S_2$ );
- (3) If the market growth rate is high and the relative market share is low then the most adequate strategy choice is divestiture strategy ( $S_3$ );
- (4) If the market growth rate is low and the relative market share is high then the most adequate strategy choice is growth strategy ( $S_4$ ).

To select a strategy for each business unit based on its position in the company's overall portfolio of businesses according to known and accepted rules could lead to different recommendations for business units placed very close to each other, on opposite sides of delimiters. To overcome such an incorrectness, we can incorporate fuzzy logic in the interpretation of the portfolio. Each business unit can be treated as a fuzzy set  $O_i$  with membership functions  $\mu_{O_i}(x_1)$  and  $\mu_{O_i}(x_2)$ . A fuzzy membership function  $\mu_{O_i}(x_k)$  assigns a real number between 0 and 1 to every parameter  $x_k$ . This number  $\mu_{O_i}(x_k)$  indicates the degree to which the characteristic  $x_k$  represents the “fuzzy set” (business unit)  $O_i$ .

#### 4.2. Fuzzy membership functions

Fuzzy membership functions can have different shapes depending on the designer's preference or experience. In our work the shapes of the membership functions were determined similarly as described in [3]. Namely, in collaboration with two domain experts, firstly the lower and the upper limits of most likely values for relevant parameters for portfolio matrix were set. The parameters relevant for the growth-share matrix are market growth rate (MGR, or input variable  $x_1$ ) with possible values from the  $R^+$  set, but the most usual values being those less than 20%, and relative market share (RMS, or input variable  $x_2$ ) that takes values most often from the interval 1.2–3.0. The values of MGR and RMS that are set as the delimiters of different quadrants in portfolio matrix are 10% and 1.5, respectively (see [1, 3, 12]). The MGR under 10% is considered, in strategic management context, as low and the values above 10% as high. Similarly, the RMS values smaller than 1.5, are low while the values greater than 1.5 are high. According to previously mentioned paradox, during the knowledge acquisition process (see [9]) experts agreed that from 6% to 20% the value of MGR can be considered more or less “high”, i.e. the degree of membership is increasing from 0 to 1. The membership function for the “low” value of fuzzy variable is decreasing from 1 in the case when MGR is less or equals 6%, to 0 for MGR values of 14% or higher. The membership function graphs of fuzzy sets “high” and “low” identified for RMS variable use two break point values (0.8 and 2.2) which were set in

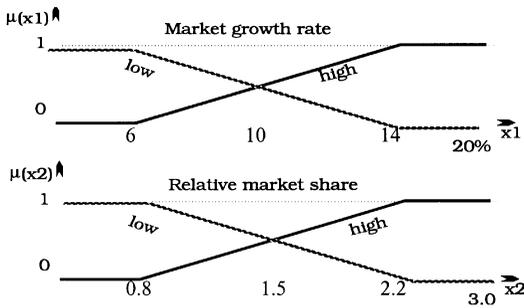


Fig. 2. Membership functions used in growth-share portfolio matrix.

consultation with domain experts. Consequently, the shapes of the membership functions were determined according to experts opinion and agreement about the suggested shapes shown in Fig. 2.

### 4.3. The fuzzy “rulebase”

The rules expressing the domain knowledge necessary for successful strategy selection, listed in Section 4.1, can be reformulated as more formal antecedent-consequent pairs or IF-THEN statements. For the growth-share matrix the production rules are the following:

IF MGR is HIGH AND RMS is HIGH  
THEN STRATEGY =  $S_1$

IF MGR is LOW AND RMS is HIGH  
THEN STRATEGY =  $S_4$

IF MGR is LOW AND RMS is LOW  
THEN STRATEGY =  $S_2$

IF MGR is HIGH AND RMS is LOW  
THEN STRATEGY =  $S_3$

### 4.4. The calculation and analyses of fit values

The inference procedure activates in parallel the antecedents of all rules. Suppose the market growth rate for business unit A equals 12% and the relative market share equals 1, 7. Let the corresponding values for business unit B be 8% and 1.3, as shown in Fig. 3. The input data pair (12, 1.7) firstly activates the rule (HIGH, HIGH;  $S_1$ ). From the fuzzy set membership function shown in Fig. 2 it can be seen that  $\mu_{\text{HIGH}}^{\text{MGR}}(12) = 0.75$  and  $\mu_{\text{HIGH}}^{\text{RMS}}(1.7) = 0.642857$ .

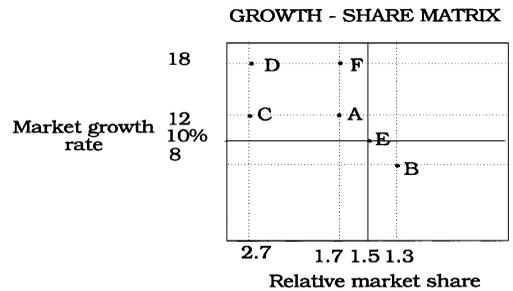


Fig. 3. The position of 6 business units in growth-share portfolio matrix.

Firstly, we combine the antecedent fit values with min operator, because the antecedent fuzzy sets are combined with conjunctive AND. These values are taken as a measure of appropriateness of strategy  $S_1$  for business unit A. If we continue the calculations, we shall see that the other strategies are less appropriate for the same business unit ( $S_2 - \min(0.25, 0.357143) = 0.25$ ;  $S_3 - \min(0.75, 0.357143) = 0.357143$ ;  $S_4 - \min(0.25, 0.642857) = 0.25$ ). The application of other operators ( $T_P, T_\lambda^{\text{SW}}, T_k^{\text{H}}$ ) lead to the same conclusion. For business units C–F, the corresponding measures of appropriateness of four strategies are shown in Tables 1–4. The exact values of relevant parameters for those business units are: C(12, 2.7), D(18, 2.7), E(10, 1.5), F(18, 1.7).

The original portfolio analyses, without the inclusion of fuzzy sets suggests for business units A, C, D and F the same strategy choice – leadership strategy. The interpretation of the same portfolio based on fuzziness recommends the leadership strategy by some degrees of adequacy and this way distinguishes their position in matrix, pointing out the possibility of pursuing “neighbor” strategy choices. The strategy choice that is the most adequate for some business unit has the greatest value whatever operator is used in the calculation. This observation is most obvious for values of  $\lambda$  in Sugeno’s family of t-norms greater than 10, and for the values of parameter  $\kappa$  in Hamacher’s t-norms not greater than  $10^3$ . To achieve similar values as for min operator, in Sugeno’s family of t-norms the value of  $\lambda$  should be greater than  $10^3$  and for Hamacher’s family  $T_k^{\text{H}}$ ,  $k$  should not exceed the value of 5. The  $T_P$ -norm gives almost equal results as the min operator (and exactly the same results as  $T_\lambda^{\text{SW}}$  when  $\lambda = 1$ ).

Table 1  
The appropriateness of strategies for six business units – using the min operator

$\min(x, y)$	A	B	C	D	E	F
S1	0.642857	0.25	0.75	1	0.5	0.642857
S2	0.25	0.642857	0	0	0.5	0
S3	0.357143	0.25	0	0	0.5	0.357146
S4	0.25	0.357143	0.25	0	0.5	0

Table 2  
The appropriateness of strategies for six business units – using the  $T_P$  operator

$T_P = T_\lambda^{SW} = T_k^H,$ $\lambda = k = 1$	A	B	C	D	E	F
S1	0.482143	0.089286	0.75	1	0.25	0.642857
S2	0.089286	0.482143	0	0	0.25	0
S3	0.267857	0.160714	0	0	0.25	0.357143
S4	0.160714	0.267857	0.25	0	0.25	0

Table 3  
The appropriateness of strategies for six business units – using the  $T_\lambda^{SW}$  operator

	A	B	C	D	E	F
$T_\lambda^{SW}, \lambda = 5$						
S1	0.467262	0.008929	0.75	1	0.208333	0.642857
S2	0.008929	0.467262	0	0	0.208333	0
S3	0.241071	0.116071	0	0	0.208333	0.357143
S4	0.116071	0.241071	0.25	0	0.208333	0
$T_\lambda^{SW}, \lambda = 1000$						
S1	0.482054	0.088804	0.75	1	0.24975	0.642857
S2	0.088804	0.482054	0	0	0.24975	0
S3	0.267697	0.160447	0	0	0.24975	0.357143
S4	0.160447	0.267697	0.25	0	0.24975	0

Table 4  
The appropriateness of strategies for six business units – using the  $T_k^H$  operator

	A	B	C	D	E	F
$T_k^H, \kappa = 0$						
S1	0.355263	0.030488	0.75	1	0.125	0.642857
S2	0.030488	0.355263	0	0	0.125	0
S3	0.163043	0.077586	0	0	0.125	0.357143
S4	0.077586	0.163043	0.25	0	0.125	0
$T_k^H, \kappa = 1000$						
S1	0.005345	0.000185	0.75	1	0.000997	0.642857
S2	0.000185	0.005345	0	0	0.000997	0
S3	0.001658	0.000598	0	0	0.000997	0.357143
S4	0.000598	0.001658	0.25	0	0.000997	0

### 5. Some other business portfolio matrices

The same situation occurs when using the industry attractiveness-business strength matrix (Fig. 4) or life cycle matrix (Fig. 5) for strategy evaluation and selection. Industry attractiveness, business unit strengths, industry maturity and competitive position, variables placed on the axes of portfolio matrices (denoted as variables  $x_3, x_4, x_5$  and  $x_6$  respectively), are expressed by fuzzy values such as high, medium, low, dominant, strong, weak, etc. Again the strategies pursued in the business unit depend on its position in portfolio matrix quadrants.

The fuzzy set values of the parameters relevant for the matrices are the following:

- industry attractiveness (IA) – high, medium or low,
- business unit strength (BUS) – high, medium or low,
- industry maturity (IM) – embryonic, growth, mature, aging,
- competitive position (CP) – dominant, strong, favorable, tenable, weak.

The use of industry attractiveness-business strength matrix first requires an identification and assessment of both critical external factors and critical internal factors of organization. After identifying and assessing these factors, top management makes a qualitative decision on whether the industry has a low, medium or high attractiveness and business unit's strength. However, despite the fact that the relevant variables for the portfolio matrix are continuous in nature, they can hardly be expressed as crisp values, but rather as subjective assessments that are vague and prone to fuzzification. For the determination of fuzzy membership functions, the qualitative assessments have to be quantized. It could be done by joining exact values or some marks or points on fixed scale or interval (similarly as in repertory grid technique) to each criterion and then summing up the points. In our example, we assumed that the maximum score can be 100 points. Fig. 6 shows membership function graphs of fuzzy sets identified for variables in portfolio matrices. Once again, the shapes of the membership functions were determined according to experts' opinion that the suggested triangular shapes are satisfactory and meaningful, and the characteristic ordinate values are well selected. At the same time, the sum of membership values is equal to 1.

		Industry attractiveness		
		high	medium	low
Business unit strengths	high	growth	growth	selective investment
	medium	growth	selective investment	harvest or divest
	low	selective investment	harvest or divest	harvest, divest or liquidate

Fig. 4. Industry attractiveness – business strengths matrix.

		Industry maturity			
		embryonic	growth	mature	aging
Competitive position	dominant				
	strong				
	favorable				
	tenable				
	weak				

Fig. 5. Life-cycle matrix.

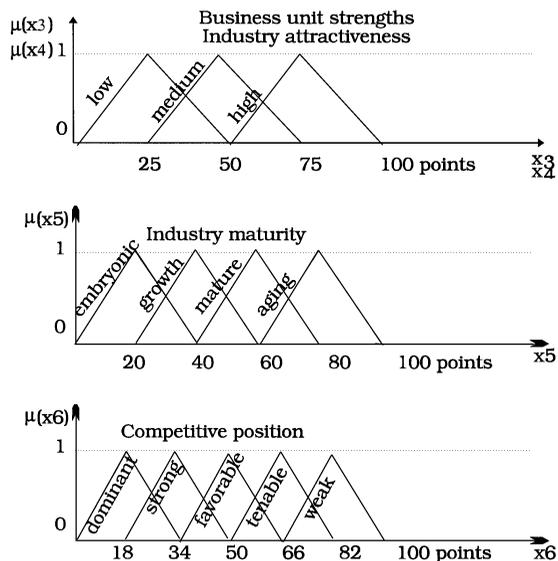


Fig. 6. Membership functions used in portfolio matrices.

Similarly, as for the growth-share matrix, production rules can be formulated for the other portfolios. For industry attractiveness-business strength matrix there will be nine rules, while for the life-cycle matrix there will be 20 rules.

Another approach to the problem of strategy selection should be the fuzzification not just of the input variables, but also the output variable, recommended strategy or strategic mix, which has discrete values. The fuzzification of strategies will be the subject of our next paper.

## 6. Conclusion

Fuzzy inference rules are very important for strategic management, in particular, for the descriptive approach that prefers intuitive, heuristic search for solutions in strategic management process. In the article we described one possible application of fuzzy sets theory in strategic management – the interpretation of portfolio matrices, used for strategy alternative(s) formulation and selection. The portfolio matrices are not exact tools, despite their apparent graphic precision. They do not give simple answers, but help rise questions and point to the need for research. The original portfolio analyses, without the inclusion of fuzzy sets suggests for business units placed in the same quadrant the same strategy choice, without taking into consideration the exact position in that quadrant. But the exact position is very important for determining the degree of adequacy of recommended strategy alternative. The interpretation of same portfolio based on fuzziness, especially the “sets as points” representation of fuzzy sets, recommends strategies by some degree of adequacy and this way points out the importance of pursuing strategy choices attached to “neighbor” quadrants. This means that the alternate strategy choices are not withdrawn, but ranked in the order that points out which ones should be examined in more depth, i.e. which questions should be paid more attention in searching for the viable answers, and

which ones could be neglected. The higher is the degree of adequacy of some generic strategy, managers should pay more attention to define better questions and talk about solutions, as these strategies are the candidates for comprising the recommended strategic mix. Consequently, the degree of adequacy should impact the priority of crafting and executing strategic moves.

The calculated degrees of adequacy for generic strategies are similar whatever operator ( $\min$ ,  $T_p$ ,  $T_\lambda^{SW}$ ,  $T_k^H$ ) was used in calculation. We think that this new approach to portfolio analyses, the incorporation of fuzzy reasoning, represents a qualitative shift in managers ability in strategic decision making.

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